```
> restart; with(plots):with(student):
Warning, the name changecoords has been redefined
```

I am not happy with this solution, I should be able to write it neater ; however, here is the code the computes the Laplace Equation in sphereical co-ords

$$X := \rho \cos(\theta) \sin(\phi)$$
$$Y := \rho \sin(\theta) \sin(\phi)$$
$$Z := \rho \cos(\phi)$$

```
> X:=rho*cos(theta)*sin(phi);
  Y:=rho*sin(theta)*sin(phi);Z:=rho*cos(phi);la1:=Diff(u,rho)*2/rho;
  La1:=(Diff(u,x)*diff(X,rho) + Diff(u,y)*diff(Y,rho)+
  Diff(u,z)*diff(Z,rho))*2/rho;
```

$$X := \rho \cos(\theta) \sin(\phi)$$
$$Y := \rho \sin(\theta) \sin(\phi)$$
$$Z := \rho \cos(\phi)$$
$$la1 := 2 \frac{\frac{\partial}{\partial \rho} u}{\rho}$$
$$La1 := 2 \frac{\left(\frac{\partial}{\partial x} u\right) \cos(\theta) \sin(\phi) + \left(\frac{\partial}{\partial y} u\right) \sin(\theta) \sin(\phi) + \left(\frac{\partial}{\partial z} u\right) \cos(\phi)}{\rho}$$

```
> Diff(u,theta);Diff(u,x)*diff(X,theta) + Diff(u,y)*diff(Y,theta)+
  Diff(u,z)*diff(Z,theta);
```

$$\frac{\partial}{\partial \theta} u$$
$$-\left(\frac{\partial}{\partial x} u\right) \rho \sin(\theta) \sin(\phi) + \left(\frac{\partial}{\partial y} u\right) \rho \cos(\theta) \sin(\phi)$$

```
> la2:=Diff(u,phi)*cot(phi)/rho^2; La2:=(Diff(u,x)*diff(X,phi) +
  Diff(u,y)*diff(Y,phi)+ Diff(u,z)*diff(Z,phi))*cot(phi)/rho^2;
```

$$la2 := \frac{\left(\frac{\partial}{\partial \phi} u\right) \cot(\phi)}{\rho^2}$$
$$La2 := \frac{\left(\left(\frac{\partial}{\partial x} u\right) \rho \cos(\theta) \cos(\phi) + \left(\frac{\partial}{\partial y} u\right) \rho \sin(\theta) \cos(\phi) - \left(\frac{\partial}{\partial z} u\right) \rho \sin(\phi)\right) \cot(\phi)}{\rho^2}$$

```
>
```

```
> la3:=Diff(Diff(u,rho),rho); La3:= (Diff(Diff(u,x),x)*diff(X,rho) +
  Diff(Diff(u,x),y)*diff(Y,rho)+
  Diff(Diff(u,x),z)*diff(Z,rho))*diff(X,rho)+(Diff(Diff(u,y),x)*diff
  (X,rho) + Diff(Diff(u,y),y)*diff(Y,rho)+
  Diff(Diff(u,y),z)*diff(Z,rho))*diff(Y,rho)+(Diff(Diff(u,z),x)*diff
  (X,rho) + Diff(Diff(u,z),y)*diff(Y,rho)+
  Diff(Diff(u,z),z)*diff(Z,rho))*diff(Z,rho)+Diff(u,x)*diff(diff(X,r
  ho),rho) +
  Diff(u,y)*diff(diff(Y,rho),rho)+Diff(u,z)*diff(diff(Z,rho),rho);si
  mplify(%,trig);
```

$$la3 := \frac{\partial^2}{\partial \rho^2} u$$

$$\begin{aligned}
La3 :=& \left(\left(\frac{\partial^2}{\partial x^2} u\right)\cos(\theta)\sin(\phi) + \left(\frac{\partial^2}{\partial y\,\partial x} u\right)\sin(\theta)\sin(\phi) + \left(\frac{\partial^2}{\partial z\,\partial x} u\right)\cos(\phi)\right)\cos(\theta)\sin(\phi) \\
&+ \left(\left(\frac{\partial^2}{\partial x\,\partial y} u\right)\cos(\theta)\sin(\phi) + \left(\frac{\partial^2}{\partial y^2} u\right)\sin(\theta)\sin(\phi) + \left(\frac{\partial^2}{\partial z\,\partial y} u\right)\cos(\phi)\right)\sin(\theta)\sin(\phi) \\
&+ \left(\left(\frac{\partial^2}{\partial x\,\partial z} u\right)\cos(\theta)\sin(\phi) + \left(\frac{\partial^2}{\partial y\,\partial z} u\right)\sin(\theta)\sin(\phi) + \left(\frac{\partial^2}{\partial z^2} u\right)\cos(\phi)\right)\cos(\phi) \\
&\cos(\theta)^2\left(\frac{\partial^2}{\partial x^2} u\right) - \cos(\theta)^2\left(\frac{\partial^2}{\partial x^2} u\right)\cos(\phi)^2 + \cos(\theta)\left(\frac{\partial^2}{\partial y\,\partial x} u\right)\sin(\theta) \\
&- \cos(\theta)\left(\frac{\partial^2}{\partial y\,\partial x} u\right)\sin(\theta)\cos(\phi)^2 + \cos(\theta)\sin(\phi)\left(\frac{\partial^2}{\partial z\,\partial x} u\right)\cos(\phi) + \sin(\theta)\left(\frac{\partial^2}{\partial x\,\partial y} u\right)\cos(\theta) \\
&- \sin(\theta)\left(\frac{\partial^2}{\partial x\,\partial y} u\right)\cos(\theta)\cos(\phi)^2 + \left(\frac{\partial^2}{\partial y^2} u\right) - \left(\frac{\partial^2}{\partial y^2} u\right)\cos(\phi)^2 - \left(\frac{\partial^2}{\partial y^2} u\right)\cos(\theta)^2 \\
&+ \left(\frac{\partial^2}{\partial y^2} u\right)\cos(\theta)^2\cos(\phi)^2 + \sin(\theta)\sin(\phi)\left(\frac{\partial^2}{\partial z\,\partial y} u\right)\cos(\phi) + \cos(\phi)\left(\frac{\partial^2}{\partial x\,\partial z} u\right)\cos(\theta)\sin(\phi) \\
&+ \cos(\phi)\left(\frac{\partial^2}{\partial y\,\partial z} u\right)\sin(\theta)\sin(\phi) + \left(\frac{\partial^2}{\partial z^2} u\right)\cos(\phi)^2
\end{aligned}$$

```
>
> la4:=Diff(Diff(u,phi),phi)/(rho^2); La4:=
  ((Diff(Diff(u,x),x)*diff(X,phi) + Diff(Diff(u,x),y)*diff(Y,phi)+
  Diff(Diff(u,x),z)*diff(Z,phi))*diff(X,phi)+(Diff(Diff(u,y),x)*diff
  (X,phi) + Diff(Diff(u,y),y)*diff(Y,phi)+
  Diff(Diff(u,y),z)*diff(Z,phi))*diff(Y,phi)+(Diff(Diff(u,z),x)*diff
  (X,phi) + Diff(Diff(u,z),y)*diff(Y,phi)+
  Diff(Diff(u,z),z)*diff(Z,phi))*diff(Z,phi)+Diff(u,x)*diff(diff(X,p
  hi),phi) +
  Diff(u,y)*diff(diff(Y,phi),phi)+Diff(u,z)*diff(diff(Z,phi),phi))/(
```

```
rho^2);simplify(%,trig);
```

$$la4 := \frac{\frac{\partial^2}{\partial \phi^2} u}{\rho^2}$$

$$La4 := \Bigg(\left(\left(\frac{\partial^2}{\partial x^2} u\right) \rho \cos(\theta) \cos(\phi) + \left(\frac{\partial^2}{\partial y\, \partial x} u\right) \rho \sin(\theta) \cos(\phi) - \left(\frac{\partial^2}{\partial z\, \partial x} u\right) \rho \sin(\phi)\right) \rho \cos(\theta) \cos(\phi)$$
$$+ \left(\left(\frac{\partial^2}{\partial x\, \partial y} u\right) \rho \cos(\theta) \cos(\phi) + \left(\frac{\partial^2}{\partial y^2} u\right) \rho \sin(\theta) \cos(\phi) - \left(\frac{\partial^2}{\partial z\, \partial y} u\right) \rho \sin(\phi)\right) \rho \sin(\theta) \cos(\phi)$$
$$- \left(\left(\frac{\partial^2}{\partial x\, \partial z} u\right) \rho \cos(\theta) \cos(\phi) + \left(\frac{\partial^2}{\partial y\, \partial z} u\right) \rho \sin(\theta) \cos(\phi) - \left(\frac{\partial^2}{\partial z^2} u\right) \rho \sin(\phi)\right) \rho \sin(\phi)$$
$$- \left(\frac{\partial}{\partial x} u\right) \rho \cos(\theta) \sin(\phi) - \left(\frac{\partial}{\partial y} u\right) \rho \sin(\theta) \sin(\phi) - \left(\frac{\partial}{\partial z} u\right) \rho \cos(\phi)\Bigg) \Big/ \rho^2$$

$$\Bigg(\cos(\theta)^2 \cos(\phi)^2 \left(\frac{\partial^2}{\partial x^2} u\right) \rho + \cos(\theta) \cos(\phi)^2 \left(\frac{\partial^2}{\partial y\, \partial x} u\right) \rho \sin(\theta)$$
$$- \cos(\theta) \cos(\phi) \left(\frac{\partial^2}{\partial z\, \partial x} u\right) \rho \sin(\phi) + \sin(\theta) \cos(\phi)^2 \left(\frac{\partial^2}{\partial x\, \partial y} u\right) \rho \cos(\theta) + \cos(\phi)^2 \left(\frac{\partial^2}{\partial y^2} u\right) \rho$$
$$- \cos(\phi)^2 \left(\frac{\partial^2}{\partial y^2} u\right) \rho \cos(\theta)^2 - \sin(\theta) \cos(\phi) \left(\frac{\partial^2}{\partial z\, \partial y} u\right) \rho \sin(\phi)$$
$$- \sin(\phi) \left(\frac{\partial^2}{\partial x\, \partial z} u\right) \rho \cos(\theta) \cos(\phi) - \sin(\phi) \left(\frac{\partial^2}{\partial y\, \partial z} u\right) \rho \sin(\theta) \cos(\phi) + \left(\frac{\partial^2}{\partial z^2} u\right) \rho$$
$$- \left(\frac{\partial^2}{\partial z^2} u\right) \rho \cos(\phi)^2 - \left(\frac{\partial}{\partial x} u\right) \cos(\theta) \sin(\phi) - \left(\frac{\partial}{\partial y} u\right) \sin(\theta) \sin(\phi) - \left(\frac{\partial}{\partial z} u\right) \cos(\phi)\Bigg) / \rho$$

```
> la5:=Diff(Diff(u,theta),theta)/(rho*sin(phi))^2; La5:=
((Diff(Diff(u,x),x)*diff(X,theta) +
Diff(Diff(u,x),y)*diff(Y,theta)+
Diff(Diff(u,x),z)*diff(Z,theta))*diff(X,theta)+(Diff(Diff(u,y),x)*
diff(X,theta) + Diff(Diff(u,y),y)*diff(Y,theta)+
Diff(Diff(u,y),z)*diff(Z,theta))*diff(Y,theta)+(Diff(Diff(u,z),x)*
diff(X,theta) + Diff(Diff(u,z),y)*diff(Y,theta)+
Diff(Diff(u,z),z)*diff(Z,theta))*diff(Z,theta)+Diff(u,x)*diff(diff
(X,theta),theta) +
Diff(u,y)*diff(diff(Y,theta),theta)+Diff(u,z)*diff(diff(Z,theta),t
heta))/(rho*sin(phi))^2;simplify(%,trig);
```

$$la5 := \frac{\frac{\partial^2}{\partial \theta^2} u}{\rho^2 \sin(\phi)^2}$$

$$La5 := \left(-\left(-\left(\frac{\partial^2}{\partial x^2} u \right) \rho \sin(\theta) \sin(\phi) + \left(\frac{\partial^2}{\partial y\, \partial x} u \right) \rho \cos(\theta) \sin(\phi) \right) \rho \sin(\theta) \sin(\phi) \right.$$
$$+ \left(-\left(\frac{\partial^2}{\partial x\, \partial y} u \right) \rho \sin(\theta) \sin(\phi) + \left(\frac{\partial^2}{\partial y^2} u \right) \rho \cos(\theta) \sin(\phi) \right) \rho \cos(\theta) \sin(\phi)$$
$$\left. - \left(\frac{\partial}{\partial x} u \right) \rho \cos(\theta) \sin(\phi) - \left(\frac{\partial}{\partial y} u \right) \rho \sin(\theta) \sin(\phi) \right) \Big/ (\rho^2 \sin(\phi)^2)$$
$$- \left(-\left(\frac{\partial^2}{\partial x^2} u \right) \rho \sin(\phi) + \left(\frac{\partial^2}{\partial x^2} u \right) \rho \sin(\phi) \cos(\theta)^2 + \sin(\theta) \left(\frac{\partial^2}{\partial y\, \partial x} u \right) \rho \cos(\theta) \sin(\phi) \right.$$
$$+ \cos(\theta) \left(\frac{\partial^2}{\partial x\, \partial y} u \right) \rho \sin(\theta) \sin(\phi) - \left(\frac{\partial^2}{\partial y^2} u \right) \rho \cos(\theta)^2 \sin(\phi) + \left(\frac{\partial}{\partial x} u \right) \cos(\theta) + \left(\frac{\partial}{\partial y} u \right) \sin(\theta)$$
$$\left. \right) / (\rho \sin(\phi))$$

```
> 
> la:=eval(la1+la2+la3+la4+la5);La :=
  eval(La1+La2+La3+La4+La5);print(la);simplify(La,trig);
```

$$la := 2 \frac{\frac{\partial}{\partial \rho} u}{\rho} + \frac{\left(\frac{\partial}{\partial \phi} u \right) \cot(\phi)}{\rho^2} + \left(\frac{\partial^2}{\partial \rho^2} u \right) + \frac{\frac{\partial^2}{\partial \phi^2} u}{\rho^2} + \frac{\frac{\partial^2}{\partial \theta^2} u}{\rho^2 \sin(\phi)^2}$$

$$La := 2 \frac{\left(\frac{\partial}{\partial x} u \right) \cos(\theta) \sin(\phi) + \left(\frac{\partial}{\partial y} u \right) \sin(\theta) \sin(\phi) + \left(\frac{\partial}{\partial z} u \right) \cos(\phi)}{\rho}$$
$$+ \frac{\left(\left(\frac{\partial}{\partial x} u \right) \rho \cos(\theta) \cos(\phi) + \left(\frac{\partial}{\partial y} u \right) \rho \sin(\theta) \cos(\phi) - \left(\frac{\partial}{\partial z} u \right) \rho \sin(\phi) \right) \cot(\phi)}{\rho^2}$$
$$+ \left(\left(\frac{\partial^2}{\partial x^2} u \right) \cos(\theta) \sin(\phi) + \left(\frac{\partial^2}{\partial y\, \partial x} u \right) \sin(\theta) \sin(\phi) + \left(\frac{\partial^2}{\partial z\, \partial x} u \right) \cos(\phi) \right) \cos(\theta) \sin(\phi)$$
$$+ \left(\left(\frac{\partial^2}{\partial x\, \partial y} u \right) \cos(\theta) \sin(\phi) + \left(\frac{\partial^2}{\partial y^2} u \right) \sin(\theta) \sin(\phi) + \left(\frac{\partial^2}{\partial z\, \partial y} u \right) \cos(\phi) \right) \sin(\theta) \sin(\phi)$$
$$+ \left(\left(\frac{\partial^2}{\partial x\, \partial z} u \right) \cos(\theta) \sin(\phi) + \left(\frac{\partial^2}{\partial y\, \partial z} u \right) \sin(\theta) \sin(\phi) + \left(\frac{\partial^2}{\partial z^2} u \right) \cos(\phi) \right) \cos(\phi) + \left(\right.$$

$$\left(\left(\frac{\partial^2}{\partial x^2}u\right)\rho\cos(\theta)\cos(\phi)+\left(\frac{\partial^2}{\partial y\,\partial x}u\right)\rho\sin(\theta)\cos(\phi)-\left(\frac{\partial^2}{\partial z\,\partial x}u\right)\rho\sin(\phi)\right)\rho\cos(\theta)\cos(\phi)$$
$$+\left(\left(\frac{\partial^2}{\partial x\,\partial y}u\right)\rho\cos(\theta)\cos(\phi)+\left(\frac{\partial^2}{\partial y^2}u\right)\rho\sin(\theta)\cos(\phi)-\left(\frac{\partial^2}{\partial z\,\partial y}u\right)\rho\sin(\phi)\right)\rho\sin(\theta)\cos(\phi)$$
$$-\left(\left(\frac{\partial^2}{\partial x\,\partial z}u\right)\rho\cos(\theta)\cos(\phi)+\left(\frac{\partial^2}{\partial y\,\partial z}u\right)\rho\sin(\theta)\cos(\phi)-\left(\frac{\partial^2}{\partial z^2}u\right)\rho\sin(\phi)\right)\rho\sin(\phi)$$
$$-\left(\frac{\partial}{\partial x}u\right)\rho\cos(\theta)\sin(\phi)-\left(\frac{\partial}{\partial y}u\right)\rho\sin(\theta)\sin(\phi)-\left(\frac{\partial}{\partial z}u\right)\rho\cos(\phi)\Bigg)\Big/\rho^2+\Bigg($$
$$-\left(-\left(\frac{\partial^2}{\partial x^2}u\right)\rho\sin(\theta)\sin(\phi)+\left(\frac{\partial^2}{\partial y\,\partial x}u\right)\rho\cos(\theta)\sin(\phi)\right)\rho\sin(\theta)\sin(\phi)$$
$$+\left(-\left(\frac{\partial^2}{\partial x\,\partial y}u\right)\rho\sin(\theta)\sin(\phi)+\left(\frac{\partial^2}{\partial y^2}u\right)\rho\cos(\theta)\sin(\phi)\right)\rho\cos(\theta)\sin(\phi)$$
$$-\left(\frac{\partial}{\partial x}u\right)\rho\cos(\theta)\sin(\phi)-\left(\frac{\partial}{\partial y}u\right)\rho\sin(\theta)\sin(\phi)\Bigg)\Big/(\rho^2\sin(\phi)^2)$$

$$2\frac{\frac{\partial}{\partial\rho}u}{\rho}+\frac{\left(\frac{\partial}{\partial\phi}u\right)\cot(\phi)}{\rho^2}+\left(\frac{\partial^2}{\partial\rho^2}u\right)+\frac{\frac{\partial^2}{\partial\phi^2}u}{\rho^2}+\frac{\frac{\partial^2}{\partial\theta^2}u}{\rho^2\sin(\phi)^2}$$
$$\left(\frac{\partial^2}{\partial x^2}u\right)+\left(\frac{\partial^2}{\partial z^2}u\right)+\left(\frac{\partial^2}{\partial y^2}u\right)$$

>