Name _____ May 3, 2004

Math. 2586h Final Exam Spring, 2004

I. Answer the following:

1.) Let S be the plane $S : x + y + z = 1$ Find the surface area of the plane in the first octant.Extar credit if you can verify it via other methods.

2.) Derive the expression for the volume of a cylinder of height h and radius a.

3.) Find the volume inside the cone $z = \sqrt{(}x^2 + y^2)$ and below the plane $z = h$.

4.) Describe the integral in spherical coordinates and evaluate it. $\int_0^{\pi} \int_0^{\frac{\pi}{2}} \int_1^2 \rho^2 \sin(\phi) d\rho \; d\phi \; d\theta$

5.) Find the volume inside both the sphere $x^2 + y^2 + z^2 = 9$ and the cylinder $x^2 + y^2 = 1$.

6.) The volume bounded by the cone whose angle in spherical coordiantes is given by $\phi = \frac{\pi}{6}$ and a sphere of radius 2.

7.) The volume bounded by the two surfaces $z = 8 - x^2 - y^2$ and $z = x^2 + 3y^2$

8. If $\Omega = (x, y) : -1 \leq x \leq +1, -1 \leq y \leq +1$ and γ the boundary of Ω.
   Set $M(x, y) = -ye^x$ and $N(x, y) = xe^y$
   Verify Green's Theorem

9. Suppose $F(x, y) = (y^2, 2xy + 1)$

   a.) Find $f(x, y)$ such that $\nabla f = F$.

   b.) Use this to evaluate $\int_a^b F \cdot dx$ where $a = (1, 1)$ & $b = (2, 3)$.

10.) For $F(x, y, z) = \dfrac{Qr}{\|r\|}$ where $r = x\imath + y\jmath + zk$

   a.) Find $\nabla \cdot F$

   b.) Find $\nabla \times F$

11.) Evaluate $\int_C y^2 dx + x^2 dy$ where C is the line from $(0, 0)$ to $(1, 1)$

12.) Evaluate $\int_C (x^2 + y^2) dx$ for the curve $x^2 + y^2 = 1$