Math 2673 3/5/2009 **Sample Test 2**

1. For $f(x,y) = x\exp(y) + \cos(xy)$ and $P = (2,0)$.
   (a) Find the rate of change in the$(3,-4)$ direction.
   (b) Find the gradient vector at P.
   (c) IN what direction is the maximum rate of change?
   (d) What is the maximum rate of change at P?
   (e) What is the direction of zero change at P, and what is that change at P?

2. For $x^2 + y^2 + z - 9 = 0$ and $P = (1,2,4)$. Find the equation of the tangent plane at P.

3. For the function. $f(x,y) = -x^2 + xy - 2y - 2x - y^2 + 4$
   (a) Find all critical points.
   (b) Classify the points as to being local maximum,local minimum values or saddle points.

4. If $w = xy + z$ and $x = \cos(t), y = \sin(t), z = t$.
   (a) use **(ONLY)** the Chain Rule to find $\dfrac{dw}{dt}$
   (b) use the **(ONLY)** Chain Rule to find $\dfrac{d^2w}{dt^2}$.

5. If $z = x^2 + y^2$, where $x = u + v$ and $y = u - v$,
   (a) Compute $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2}$.
   (b) use the Chain Rule to find $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial^2 z}{\partial u^2}$

6. Show $f(x,y,z) = e^{3x+4y} * \cos(5z)$ satisfies $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} + \dfrac{\partial^2 f}{\partial z^2} = 0$

7. For the function. $f(x,y) = -x^2 + xy - 2y - 2x - y^2 + 4$

(a) Find all critical points.

(b) Classify the points as to being local maximum,local minimum values or saddle points.

8. For the intersection of the surface $z = x^2 + y^2$ and the plane $z = 1$ find the rate of change of the tangent line to the curve of at the point $(a, b, 1)$.

9. Find the all critical points for the following function and label them as local maximum and minimum values and saddle points accordingly where
$f(x, y) = 4xy - x^4 - y^4 + 1$

10. Find the equation of the tangent plane at the point $P = (-3, 1, -3)$ to the ellipsoid $x^2/9 + y^2 + z^2/9 = 3$

11. For $u = x^4 * y + y^2 * z^3$ and $x = r * s * e^t$, $y = r * s^2 * e^{-t}$ and $z = r * s * \sin(t)$ find $\dfrac{\partial u}{\partial s}$.

12. Temperature is given by $T(x, y, z) = \dfrac{80}{1 + x^2 + 2y^2 + 3z^2}$ where $\bigtriangledown T = \dfrac{\partial T}{\partial x}i + \dfrac{\partial T}{\partial y}j + \dfrac{\partial T}{\partial z}k$

    (a) In what direction does T increase most rapidly?

    (b) What is that increase?

13. Use Lagrange multipliers to find the dimensions of the maximum rectangular box with "no lid" which is made from $12ft^2$ of cardboard.

14. For $z = x^2 + 3xy - y^2$ find the tangent plane at (2,3).

    (a) Use the this to approximate the value at $(2.05, 2.96)$

    (b) What is the error?