```
> restart:with(plots):with(student):
```

```
>   plot1:=implicitplot3d(x^2 + y^2 + (z-1)^2 = 1, x = -1..1,y =
-1 .. 1, z = 0 .. 2,color=yellow):
display3d({plot1}, axes=boxed):plot3d(1+sqrt(1-r^2), theta=0..2*
Pi,  r= -1..1, coords=cylindrical,color = red, axes=boxed)
:plot3d(2*cos(phi), theta=0..2*Pi, phi=0..Pi, coords=spherical,
color = green, axes=boxed):
```

Find volume in rect

```
> 4* Int ( Int ( Int ( 1, z = 1 - sqrt(1 -x^2 - y^2) .. 1 + sqrt(1
- x^2 - y^2) ), y = 0 .. sqrt(1-x^2) ),x = 0.. 1 ) ;
```

$$4\left(\int_0^1\int_0^{\sqrt{1-x^2}}\int_{1-\sqrt{1-x^2-y^2}}^{1+\sqrt{1-x^2-y^2}} 1\,dz\,dy\,dx\right) \tag{1}$$

```
> ?
```

```
> ?
```

```
> 4* int ( int ( int ( 1, z = 1 - sqrt(1 -x^2 - y^2) .. 1 + sqrt(1
- x^2 - y^2) ), y = 0 .. sqrt(1-x^2) ),x = 0.. 1 ) ; # answer in
rects
```

$$\frac{4}{3}\pi \tag{2}$$

so try polars/cylindricals

```
> 4* Int ( Int ( Int ( r, z = 1 - sqrt(1 -r^2 ) .. 1 + sqrt(1 -
r^2 ) ), r = 0 .. 1 ),theta  = 0.. Pi/2 ) ;
> 4* int ( int ( int ( r, z = 1 - sqrt(1 -r^2 ) .. 1 + sqrt(1 -
r^2 ) ), r = 0 .. 1 ),theta  = 0.. Pi/2 ) ;
```

$$4\left(\int_0^{\frac{1}{2}\pi}\int_0^1\int_{1-\sqrt{1-r^2}}^{1+\sqrt{1-r^2}} r\,dz\,dr\,d\theta\right) \tag{3}$$

$$\frac{4}{3}\pi$$

```
> 4* Int ( Int ( Int ( rho^2*sin(phi), rho = 0 .. 2*cos(phi) ),
phi = 0 .. Pi/2 ),theta  = 0.. Pi/2 ) ;
```

$$4\left(\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\int_0^{2\cos(\phi)} \rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta\right) \tag{4}$$

```
> ?
> 4* int ( int ( int ( rho^2*sin(phi), rho = 0 .. 2*cos(phi)), phi
   = 0 .. Pi/2 ),theta  = 0.. Pi/2 ) ;
```

$$\frac{4}{3}\pi \tag{5}$$

```
> # sno cone p 1045 ex 4
> ?
> restart:with(plots):with(student):

> p1:=implicitplot3d(x^2 + y^2 + z^2 = z, x = 0..1,y = 0 .. 1, z =
  0 .. 2,color=yellow, axes = boxed):

> ?
> ?
> ?
> p2 :=implicitplot3d(sqrt(x^2 + y^2) = z, x = -1..1,y = -1 .. 1,
  z = 0 .. 1,color= red, axes = boxed):
> ?
> display3d(p1,p2):

> # volume in rect
> 4* Int ( Int ( Int ( 1, z =  sqrt(x^2 + y^2) ..  sqrt(1 - x^2 -
  y^2) ), y = 0 .. sqrt(1-x^2) ),x = 0.. 1 ) ;
```

$$4\left(\int_0^1\int_0^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{1-x^2-y^2}} 1\,dz\,dy\,dx\right) \tag{6}$$

```
> ?

> ?

> 4* int ( int ( int ( 1, z =  sqrt(x^2 + y^2) .. 1 + sqrt(1 - x^2
  - y^2) ), y = 0 .. sqrt(1-x^2) ),x = 0.. 1 ) : # answer in rects
Warning, unable to determine if (-x^2)^(1/2) is between 0 and (1-x^2)^(1/2);
try to use assumptions or set _EnvAllSolutions to true
Warning, unable to determine if -(-x^2)^(1/2) is between 0 and (1-x^2)^(1/2);
try to use assumptions or set _EnvAllSolutions to true
Warning, unable to determine if -(1-x^2)^(1/2) is between 0 and (1-x^2)^(1/2)
; try to use assumptions or set _EnvAllSolutions to true
> # volume in spherical

> q1 :=plot3d(2*cos(phi), theta=0..2*Pi, phi=0..Pi, coords=
  spherical,color = green, axes=boxed):
```

```
> q2:=implicitplot3d(phi=Pi/4, rho = 0 ..2,theta=0..2*Pi, phi=0..
  Pi, coords=spherical,color = red, axes=boxed):
> display3d(q1,q2):
> 4* Int ( Int ( Int ( rho^2*sin(phi), rho = 0 .. cos(phi) ), phi
  = 0 .. Pi/4 ),theta  = 0.. Pi/2 ) ;
```

$$4\left(\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{4}\pi}\int_0^{\cos(\phi)}\rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta\right) \tag{7}$$

```
> ?
> 4* int ( int ( int ( rho^2*sin(phi), rho = 0 .. cos(phi)), phi
  = 0 .. Pi/4 ),theta  = 0.. Pi/2 ) ;
```

$$\frac{1}{8}\pi \tag{8}$$

```
> #  cone p
> ?
> restart:with(plots):with(student):

> p1:=implicitplot3d(1 = z, x = -1..1,y = -1 .. 1, z = 0 .. 2,
  color=yellow, axes = boxed):

> ?
> ?
> ?
> p2 :=implicitplot3d(sqrt(x^2 + y^2) = z, x = -1..1,y = -1 .. 1,
  z = 0 .. 1,color= red, axes = boxed):
> ?
> display3d(p1,p2):
> # volume in rect
> 4* Int ( Int ( Int ( 1, z =  sqrt(x^2 + y^2) ..  1 ), y = 0 ..
  sqrt(1-x^2) ),x = 0.. 1 ) ;
```

$$4\left(\int_0^1\int_0^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^1 1\,dz\,dy\,dx\right) \tag{9}$$

```
> ?

> ?

> 4* int ( int ( int ( 1, z =  sqrt(x^2 + y^2) .. 1  ), y = 0 ..
  sqrt(1-x^2) ),x = 0.. 1 ) ; # no answer in rects
```

(10)

$$4\left(\int_0^1\left(\frac{1}{4}x^2\ln(x^2)-\frac{1}{2}x^2\ln\left(\sqrt{1-x^2}+1\right)+\frac{1}{2}\sqrt{1-x^2}\right)dx\right) \quad (10)$$

```
> 4* Int ( Int ( Int ( rho^2*sin(phi), rho = 0 .. sec(phi) ), phi
  = 0 .. Pi/4 ),theta  = 0.. Pi/2 ) ;
```

$$4\left(\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{4}\pi}\int_0^{\sec(\phi)}\rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta\right) \quad (11)$$

```
> ?
```

```
> 4* int ( int ( int ( rho^2*sin(phi), rho = 0 .. sec(phi)), phi
  = 0 .. Pi/4 ),theta  = 0.. Pi/2 ) ;
```

$$\frac{1}{3}\pi \quad (12)$$