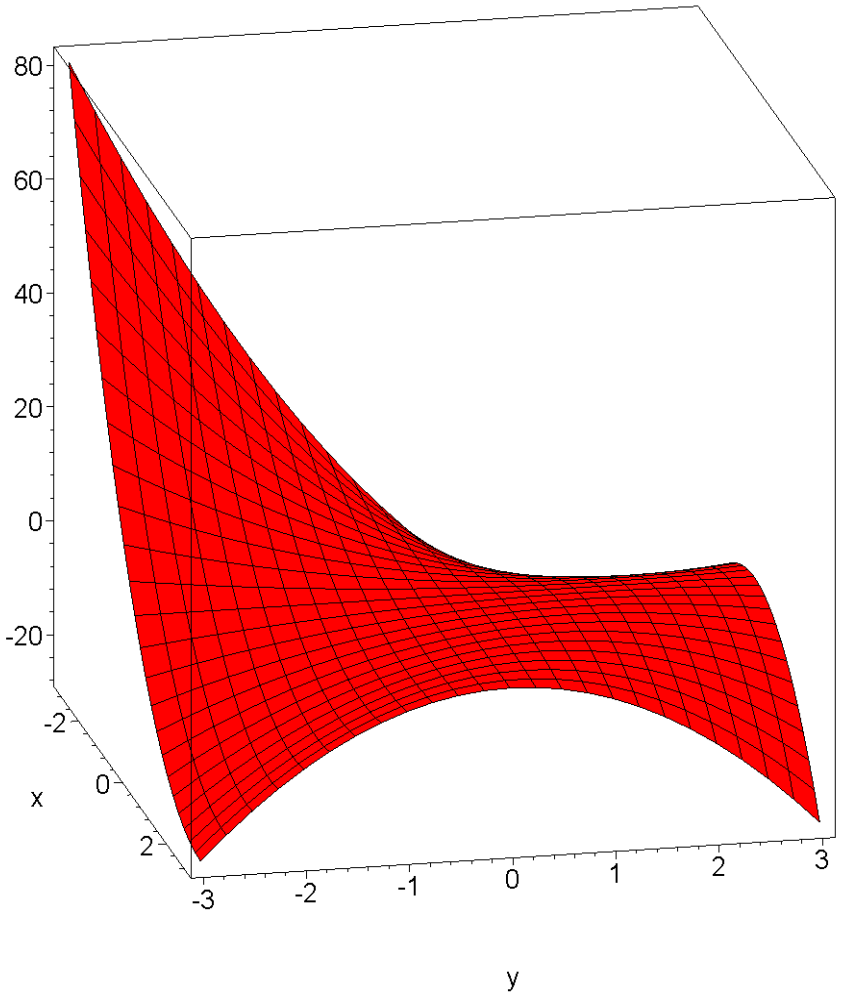
```
> restart:with(Student[VectorCalculus]):with( plots): with(student):
Warning, the assigned names <,> and <|> now have a global binding

Warning, these protected names have been redefined and unprotected: *, +, -, ., D,
Vector, diff, int, limit, series

Warning, the name changecoords has been redefined

Warning, the protected name D has had its previous binding removed and has been
assigned
```

```
> # Ex 1
>
```

```
>  f:= (x,y) -> 3*x*y-y*x^2-x*y^2;plot3d(f(x,y), x = -3..3,y=-3 ..
  3,
  color=red,axes=boxed);fxx:=diff(3*x*y-y*x^2-x*y^2,x,x);fx:=diff(3*
  x*y-y*x^2-x*y^2,x);fy:=diff(3*x*y-y*x^2-x*y^2,y);H:=
  diff(3*x*y-y*x^2-x*y^2,x,x)*diff(3*x*y-y*x^2-x*y^2,y,y)-
  diff(3*x*y-y*x^2-x*y^2,x,y)*diff(3*x*y-y*x^2-x*y^2,x,y);
>
```

$$f := (x, y) \to Student\text{:-}VectorCalculus\text{:-}`+`(Student\text{:-}VectorCalculus\text{:-}`+`($$
$$Student\text{:-}VectorCalculus\text{:-}`*`(Student\text{:-}VectorCalculus\text{:-}`*`(3, x), y),$$
$$Student\text{:-}VectorCalculus\text{:-}`-`(Student\text{:-}VectorCalculus\text{:-}`*`(y, x^2))),$$
$$Student\text{:-}VectorCalculus\text{:-}`-`(Student\text{:-}VectorCalculus\text{:-}`*`(x, y^2)))$$



$$fxx := -2\,y$$

$$fx := 3\,y - 2\,x\,y - y^2$$

$$fy := 3\,x - x^2 - 2\,x\,y$$

$$H := 4\,x\,y - (3 - 2\,x - 2\,y)^2$$

```
> # solve fx;=0
> =solve( {fx = 0,fy=0}, [x, y]);
```

$$RXY := [[x = 0, y = 0], [x = 3, y = 0], [x = 0, y = 3], [x = 1, y = 1]]$$

```
> rx := array(1..4):ry := array(1..4):

  rx:=[0,3,0,1];ry:=[0,0,3,1];
```

$$rx := [0, 3, 0, 1]$$

$$ry := [0, 0, 3, 1]$$

```
> for i from 1 to 4 do
  h:=subs(x=rx[i],y=ry[i],H);valfxx:=subs(x=rx[i],y=ry[i],fxx);print
  (i,rx[i],ry[i],valfxx,h); end do:
```

1, 0, 0, 0, -9

2, 3, 0, 0, -9

3, 0, 3, -6, -9

4, 1, 1, -2, 3

```
> # (1,1) is a local max all other points are saddle points
> restart:with(Student[VectorCalculus]):with( plots): with(student):
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assigned

> # Ex 2

>

>  f:= (x,y) -> 4*x*y-x^4-y^4;plot3d(f(x,y), x = -3..3,y=-3 .. 3,
  color=red,axes=boxed);fxx:=diff(4*x*y-x^4-y^4,x,x);fx:=diff(4*x*y-
  x^4-y^4,x);fy:=diff(4*x*y-x^4-y^4,y);H:=
  diff(4*x*y-x^4-y^4,x,x)*diff(4*x*y-x^4-y^4,y,y)-
  diff(4*x*y-x^4-y^4,x,y)*diff(4*x*y-x^4-y^4,x,y);
>
```
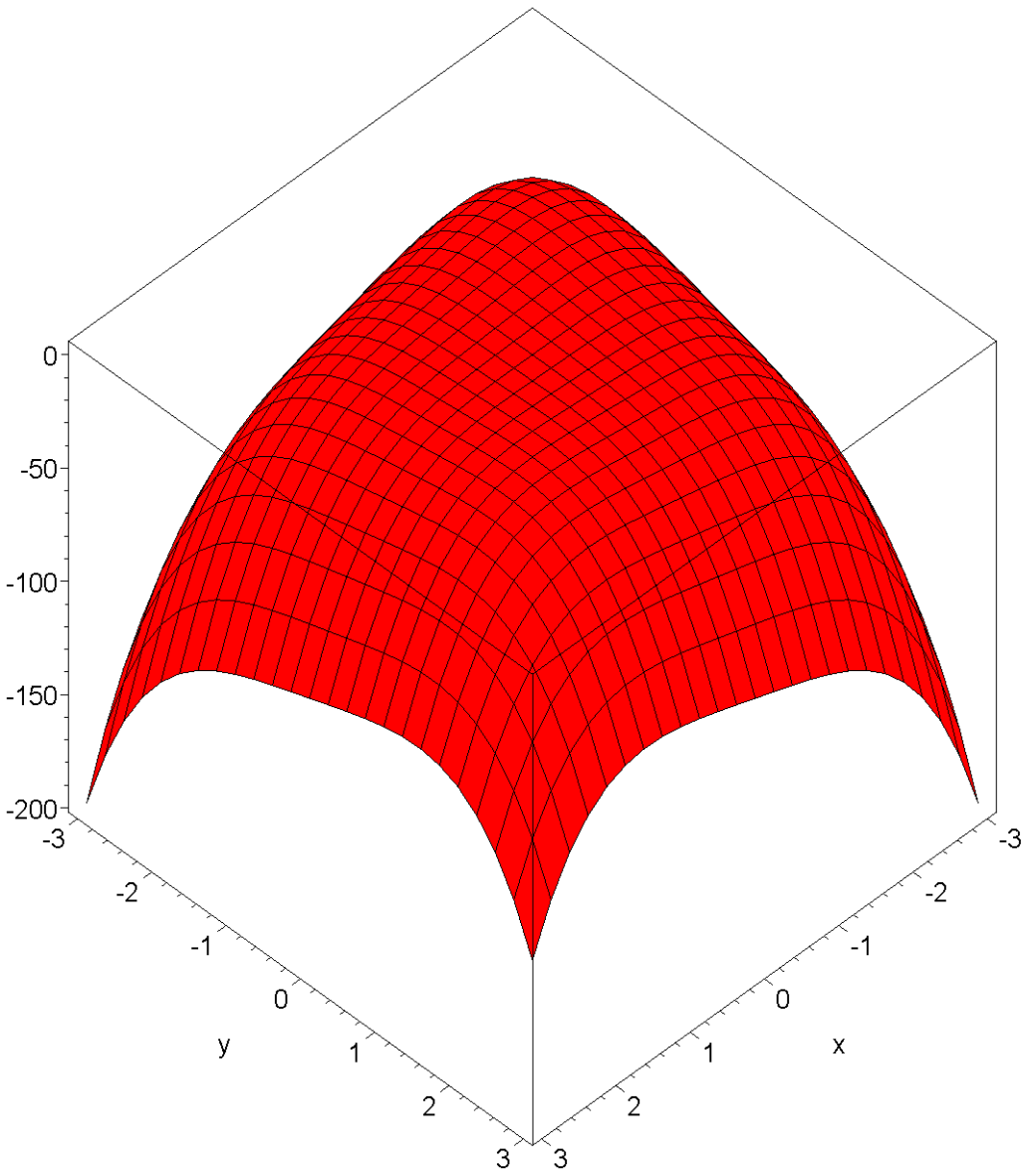

$$f := (x, y) \to Student{:}\text{-}VectorCalculus{:}\text{-}`+`(Student{:}\text{-}VectorCalculus{:}\text{-}`+`(Student{:}\text{-}VectorCalculus{:}\text{-}`*`(Student{:}\text{-}VectorCalculus{:}\text{-}`*`(4, x), y), Student{:}\text{-}VectorCalculus{:}\text{-}`-`(x^4)), Student{:}\text{-}VectorCalculus{:}\text{-}`-`(y^4))$$



$$fxx := -12\,x^2$$

$$fx := 4\,y - 4\,x^3$$

$$fy := 4\,x - 4\,y^3$$

$$H := 144\,x^2\,y^2 - 16$$

```
> # solve fx;=0
> solve( {fx = 0,fy=0}, [x, y]);
```

$$[[x = 0, y = 0], [x = \text{RootOf}(\_Z^2 + 1, label = \_L4), y = -\text{RootOf}(\_Z^2 + 1, label = \_L4)],$$
$$[x = 1, y = 1], [x = -1, y = -1], [x = \text{RootOf}(-\text{RootOf}(\_Z^2 + 1, label = \_L3) + \_Z^2, label = \_L5),$$
$$y = \text{RootOf}(-\text{RootOf}(\_Z^2 + 1, label = \_L3) + \_Z^2, label = \_L5)\ \text{RootOf}(\_Z^2 + 1, label = \_L3)]]$$

```
> rx := array(1..3):ry := array(1..3):

  rx:=[0,1,-1];ry:=[0,1,-1];
```

$$rx := [0, 1, -1]$$

$$ry := [0, 1, -1]$$

```
> for i from 1 to 3 do
  h:=subs(x=rx[i],y=ry[i],H);valfxx:=subs(x=rx[i],y=ry[i],fxx);print
  (i,rx[i],ry[i],valfxx,h); end do:
```

$$1, 0, 0, 0, -16$$

$$2, 1, 1, -12, 128$$

$$3, -1, -1, -12, 128$$

```
> # (1,1) and (-1,-1) are local max all other points are saddle
  points
> restart:with(Student[VectorCalculus]):with( plots): with(student):
Warning, the assigned names <,> and <|> now have a global binding

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> #ex8
> f:= (x,y) -> x + 2*y + 3*z=6;implicitplot3d(f(x,y), x = 0..6,y=0
  .. 3,z=0 .. 2, color=red,axes=boxed);
```
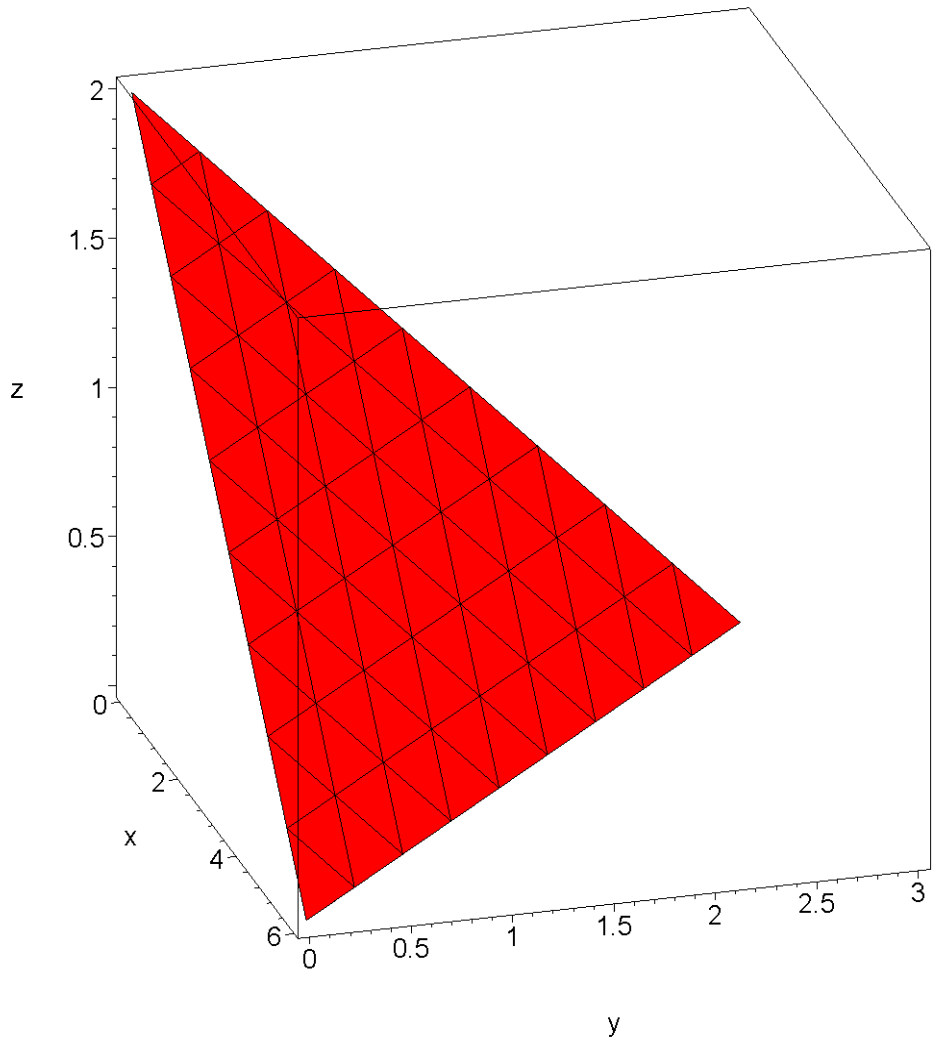

$$f := (x, y) \to Student\text{:-}VectorCalculus\text{:-}`+`($$
$$Student\text{:-}VectorCalculus\text{:-}`+`(x, Student\text{:-}VectorCalculus\text{:-}`*`(2, y)),$$
$$Student\text{:-}VectorCalculus\text{:-}`*`(3, z)) = 6$$




```
> Int(Int(Int( 1,
  z=0..(6-x-2*y)/3),y=0..(6-x)/2),x=0..6)=int(int(int( 1,
  z=0..(6-x-2*y)/3),y=0..(6-x)/2),x=0..6);
```

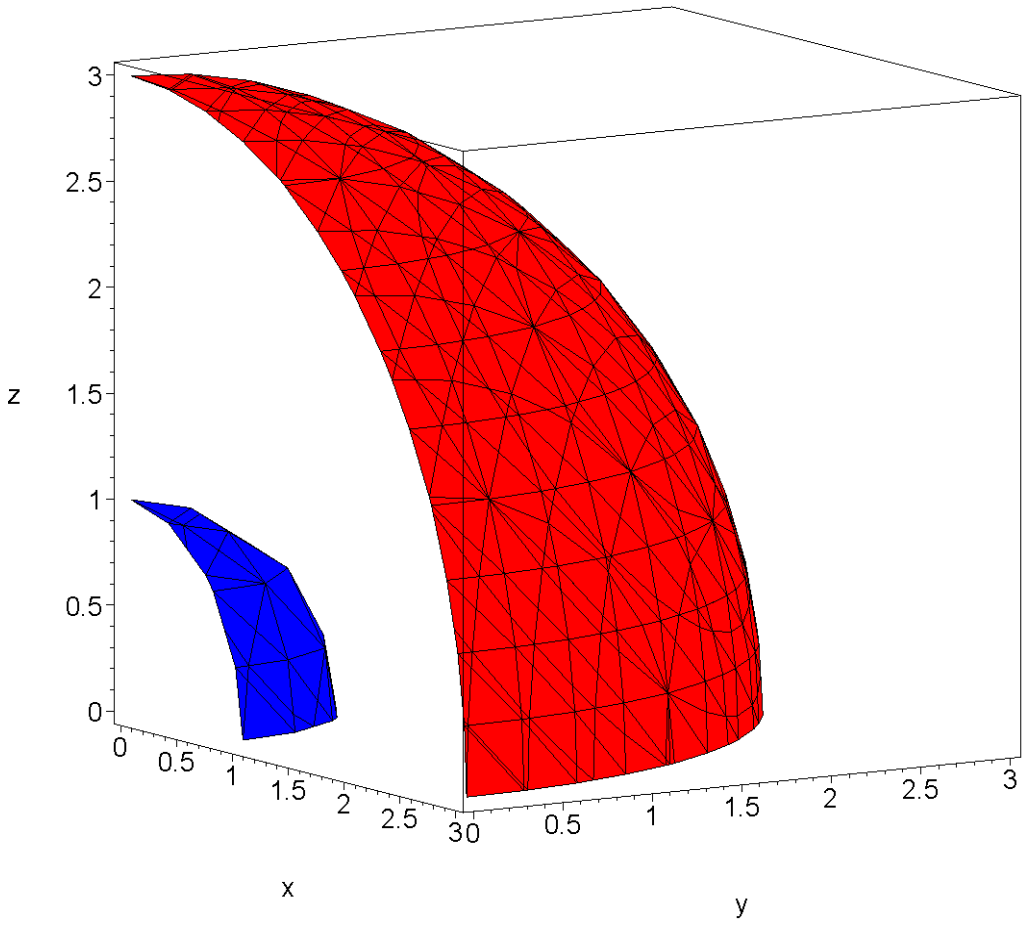
$$\int_0^6 \int_0^{3-\frac{x}{2}} \int_0^{2-\frac{x}{3}-\frac{2y}{3}} 1 \, dz \, dy \, dx = 6$$

```
>
> #ex9
> f:= (x,y) -> x^2 + y^2 + z^2=9;S1:=implicitplot3d(f(x,y), x =
  0..3,y=0 .. 3,z=0 .. 3, color=red,axes=boxed):g:= (x,y) -> x^2 +
```

```
y^2 + z^2=1;S2:=implicitplot3d(g(x,y), x = 0..3,y=0 .. 3,z=0 .. 3,
color=blue,axes=boxed):display3d(S1,S2);
```

$$f := (x, y) \to Student\text{:-}VectorCalculus\text{:-}`+`(Student\text{:-}VectorCalculus\text{:-}`+`(x^2, y^2), z^2) = 9$$

$$g := (x, y) \to Student\text{:-}VectorCalculus\text{:-}`+`(Student\text{:-}VectorCalculus\text{:-}`+`(x^2, y^2), z^2) = 1$$




```
> Int(Int(Int( 1,
z=0..sqrt(9-x^2-y^2)),y=0..sqrt(9-x^2)),x=0..3)=int(int(int( 1,
z=0..sqrt(9-x^2-y^2)),y=0..sqrt(9-x^2)),x=0..3);# the red one
```

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2-y^2}} 1 \, dz \, dy \, dx = \frac{9\,\pi}{2}$$

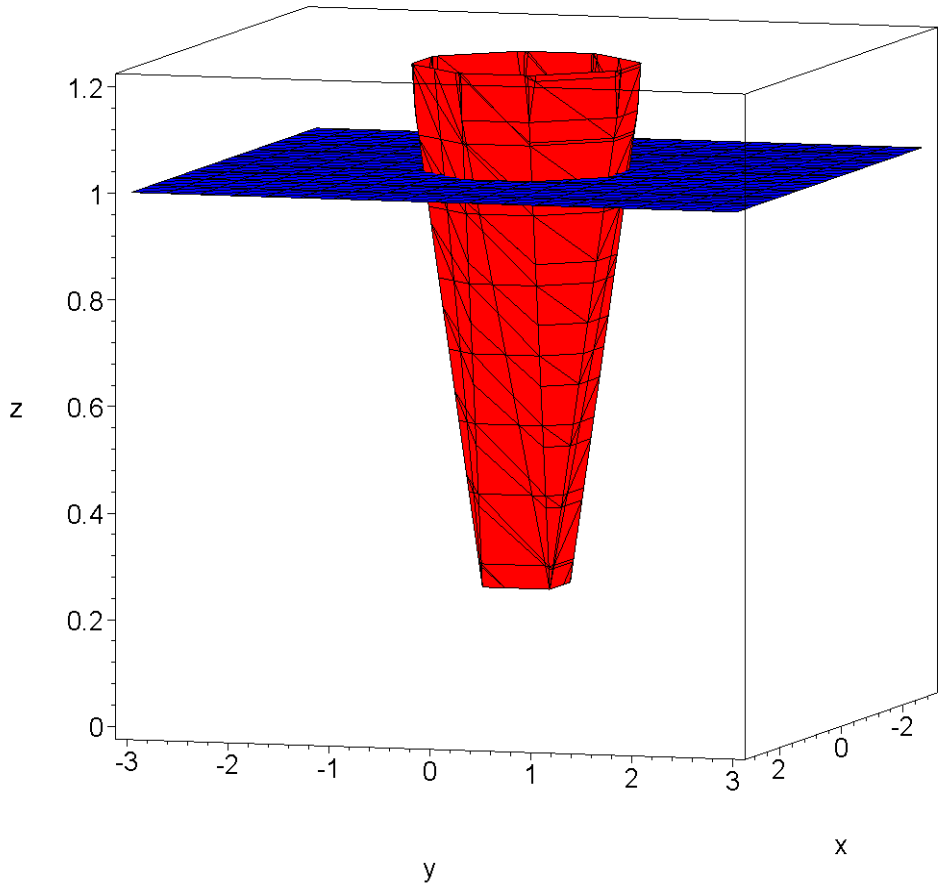
```
> Int(Int(Int( 1,
  z=0..sqrt(1-x^2-y^2)),y=0..sqrt(1-x^2)),x=0..3)=int(int(int( 1,
  z=0..sqrt(1-x^2-y^2)),y=0..sqrt(1-x^2)),x=0..3);# the blue 1
```

$$\int_0^3 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} 1 \, dz \, dy \, dx = \frac{11\,\pi}{6}$$

```
>
> #ex10
> f:= (x,y) -> x^2 + y^2 -z=0;S1:=implicitplot3d(f(x,y), x =
  -3..3,y=-3 .. 3,z=0 .. 1.2, color=red,axes=boxed):g:= (x,y) ->
  z=1;S2:=implicitplot3d(g(x,y), x = -3..3,y=-3 .. 3,z=0 .. 1.2,
  color=blue,axes=boxed):display3d(S1,S2);
```

$$f := (x, y) \to \mathit{Student}\text{:-}\mathit{VectorCalculus}\text{:-`+`}(\mathit{Student}\text{:-}\mathit{VectorCalculus}\text{:-`+`}(x^2, y^2), \mathit{Student}\text{:-}\mathit{VectorCalculus}\text{:-`-`}(z)) = 0$$

$$g := (x, y) \to z = 1$$




```
> Int(Int(Int( 1,
  z=x^2+y^2..1),y=0..sqrt(1-x^2)),x=0..1)=int(int(int( 1,
  z=x^2+y^2..1),y=0..sqrt(1-x^2)),x=0..1);
```

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{x^2+y^2}^1 1 \, dz \, dy \, dx = \frac{\pi}{8}$$

```
>
```

```
>
```