```
> restart:with(plots):with(student):with(VectorCalculus):
Warning, the name changecoords has been redefined

Warning, the assigned names <,> and <|> now have a global binding

Warning, these protected names have been redefined and unprotected: *, +, -, ., D,
Vector, diff, int, limit, series

> # problem 1

>

> Int(Int(Int(rho^2*sin(phi), rho= 1 .. 2) ,phi=0 .. Pi), theta=0
  ..2*Pi)=int(int(int(rho^2*sin(phi), rho= 1 .. 2 ),phi=0 .. Pi),
  theta=0 ..2*Pi);
```

$$\int_0^{2\pi}\int_0^{\pi}\int_1^{2}\rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta=\frac{28\pi}{3}$$

```
>
> # Ex2
> Int(Int(Int(rho^2*sin(phi), rho= 0 .. 2) ,phi=0 .. Pi/3), theta=0
  ..2*Pi)=int(int(int(rho^2*sin(phi), rho= 0 .. 2 ),phi=0 .. Pi/3),
  theta=0 ..2*Pi);
>
```

$$\int_0^{2\pi}\int_0^{\frac{\pi}{3}}\int_0^{2}\rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta=\frac{8\pi}{3}$$

```
> #Ex3
> SetCoordinates( 'cartesian'[x,y] );
> LineInt( VectorField( <x-y,x+y> ),  LineSegments( <0,0>,
  <1,0>,<0,1>,<0,0>),inert )=LineInt( VectorField( <x-y,x+y> ),
  LineSegments( <0,0>, <1,0>,<0,1>,<0,0>) );
> # now Greens' theorem
```

$$\int_0^{1}t\,dt+\int_0^{1}2\,t\,dt+\int_0^{1}-1+t\,dt=1$$

```
> Int(Int(2,y=0..1-x),x=0 ..1)=int(int(2,y=0..1-x),x=0 ..1);
```

$$\int_0^{1}\int_0^{1-x}2\,dy\,dx=1$$

```
> #Ex 4
> PathInt(x +(y), [x,y] = Path(<(t),(t)>,t=0 .. 1 ),inert ) +
  PathInt(x+(y), [x,y] = Line( <1,1>, <2,3> ),inert )=PathInt(x +
  eval(y), [x,y] = Path(<t,t>,t=0 .. 1 ),inert ) + PathInt(x+(y),
  [x,y] = Line( <1,1>, <2,3> ) );evalf(%);
```

$$\int_0^1 2\,t\sqrt{2}\,dt + \int_0^1 (2+3\,t)\sqrt{5}\,dt = \int_0^1 2\,t\sqrt{2}\,dt + \frac{7\sqrt{5}}{2}$$

$$9.240451483 = 9.240451482$$

```
>
>
> #EX5
>
>
> LineInt( VectorField( <y,x> ), Path( <cos(t),sin(t)>, t=0..Pi
  ),inert )+ LineInt( VectorField( <y,x> ), LineSegments( <-1,0>,
  <1,0>),inert )= (LineInt( VectorField( <y,x> ), Path(
  <cos(t),sin(t)>, t=0..Pi ) )+ LineInt( VectorField( <y,x> ),
  LineSegments( <1,0>, <1,0>) ));
```

$$\int_0^{\pi} -\sin(t)^2 + \cos(t)^2\,dt + \int_0^1 0\,dt = 0$$

```
> # or it is exact around a closed interval so it is zero
> # EX6
> SetCoordinates( 'cartesian'[x,y,z] );
```

$$\text{cartesian}_{x,\,y,\,z}$$

```
> v := VectorField( <y*z,x*z,x*y> );
```

$$v := y\,z\,\underline{e}_x + x\,z\,\underline{e}_y + x\,y\,\underline{e}_z$$

```
> ScalarPotential( v );
```

$$y\,z\,x$$

```
> # EX7a
> F := VectorField( <sin(x),sin(y),x*z^2> );
```

$$F := \sin(x)\,\underline{e}_x + \sin(y)\,\underline{e}_y + x\,z^2\,\underline{e}_z$$

```
> Divergence( F );
```

$$\cos(x) + \cos(y) + 2\,x\,z$$

```
> Gradient(%);
```

$$(-\sin(x) + 2\,z)\,\underline{e}_x - \sin(y)\,\underline{e}_y + 2\,x\,\underline{e}_z$$

```
> # EX7b
```

```
> Curl( F );
```

$$-z^2 e^{-y}$$

```
> Curl(%);
```

$$2\,z\,e^{-x}$$

```
> # EX8
> restart:with(plots):with(student):Int(Int(x+y, y=x-2 .. x),x=0
  ..3)= int(int(x+y,y=x-2 .. x), x=0 ..3);# change in var is x=u+v
  and y=v which gives the following region of integration then x=v
  and y= (u+v)/2
Warning, the name changecoords has been redefined
```

$$\int_0^3 \int_{x-2}^{x} x + y \, dy \, dx = 12$$

```
> Int(Int( u+2*v, u = -2 ..  0), v = 0.. 3) = int(int( u+2*v, u = -2
  ..  0), v = 0 .. 3);
```

$$\int_0^3 \int_{-2}^{0} u + 2\,v \, du \, dv = 12$$

```
> # compute the Jacobian
```

Change coordinate systems.

```
> with(VectorCalculus):(M,d):=Jacobian( [(u+v),v], [u,v],
  'determinant' );
Warning, the assigned names <,> and <|> now have a global binding

Warning, these protected names have been redefined and unprotected: *, +, -, ., D,
Vector, diff, int, limit, series
```

$$M, d := \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, 1$$

```
> # jacobian is 1
>
>
>
>
```