```
> restart:with(plots):
  with(student):with(linalg):Digits := 4:
Warning, the name changecoords has been redefined

Warning, the protected names norm and trace have been redefined and unprotected
```

sample questions First Test, Calculus 2673, 9/20/2005

1.

```
> a:=[1,3,-3]:b:=[2,2,4]:
```

i.

```
> ans1 := dotprod(a,b)/norm(a,2);
```

$$ans1 := -\frac{4}{19}\sqrt{19}$$

ii

```
> u:=a/norm(a,2); ans2 := ans1 * u;
```

$$u := \frac{1}{19}[1, 3, -3]\sqrt{19}$$

$$ans2 := \left[\frac{-4}{19}, \frac{-12}{19}, \frac{12}{19}\right]$$

2. express b as b1 + b2 where b1 is parallel to a

```
> a:=[1,3,-3];b:=[-4,1,-2];
```

$$a := [1, 3, -3]$$

$$b := [-4, 1, -2]$$

```
> u:= a/(norm(a,2));
```

$$u := \frac{1}{19}[1, 3, -3]\sqrt{19}$$

```
> b1:= dotprod(b,u)*u; b2:=b-b1;
```

$$b1 := \left[\frac{5}{19}, \frac{15}{19}, \frac{-15}{19}\right]$$

$$b2 := \left[\frac{-81}{19}, \frac{4}{19}, \frac{-23}{19}\right]$$

3. Find the equation of the plane .

```
> a:=[0,2,3];b:=[2,1,0]; c:=[1,-1,4];#  points in the  plane
```

$$a := [0, 2, 3]$$

$$b := [2, 1, 0]$$

$$c := [1, -1, 4]$$

```
> ba := b-a; bc:= b-c;  n:=crossprod(ba,bc); # normal of 2 vectors
  in the plane
```

$$ba := [2, -1, -3]$$

$$bc := [1, 2, -4]$$

$$n := [10, 5, 5]$$

```
> R:=vector([x,y,z]);
```

$$R := [x, y, z]$$

```
> u:=dotprod(R,n);k :=(dotprod(n,a)); # direction and konstant
```

$$u := 10x + 5y + 5z$$

$$k := 25$$

```
> u = k; # equation of the  plane
```

$$10x + 5y + 5z = 25$$

4. Find the equation of the plane .

```
Warning, the protected names norm and trace have been redefined and unprotected
```

```
> p:=[1,0,-1]; #  point in the  plane
```

$$p := [1, 0, -1]$$

```
> n:=[1,1,-1]; # normal vector of plane
```

$$n := [1, 1, -1]$$

```
> R:=vector([x,y,z]);
```

$$R := [x, y, z]$$

```
> u:=dotprod(R,n);k :=(dotprod(n,p));
```

$$u := x + y - z$$

$$k := 2$$

```
> u=k; # equation of  plane
```

$$x + y - z = 2$$

5 find the line

```
> p:=[1,2,3];
> v:=[1,-1,1];
```

$$p := [1, 2, 3]$$

$$v := [1, -1, 1]$$

```
> l:=evalm([1,2,3]+t*v);
```

$$l := [1 + t, 2 - t, 3 + t]$$

0.

6. Find line parralel to y-axis through [1,7,0]

```
>
> v:=[0,1,0]; #  vectors of line
```

$$v := [0, 1, 0]$$

```
> l:=evalm([1,7,0]+t*v);
```

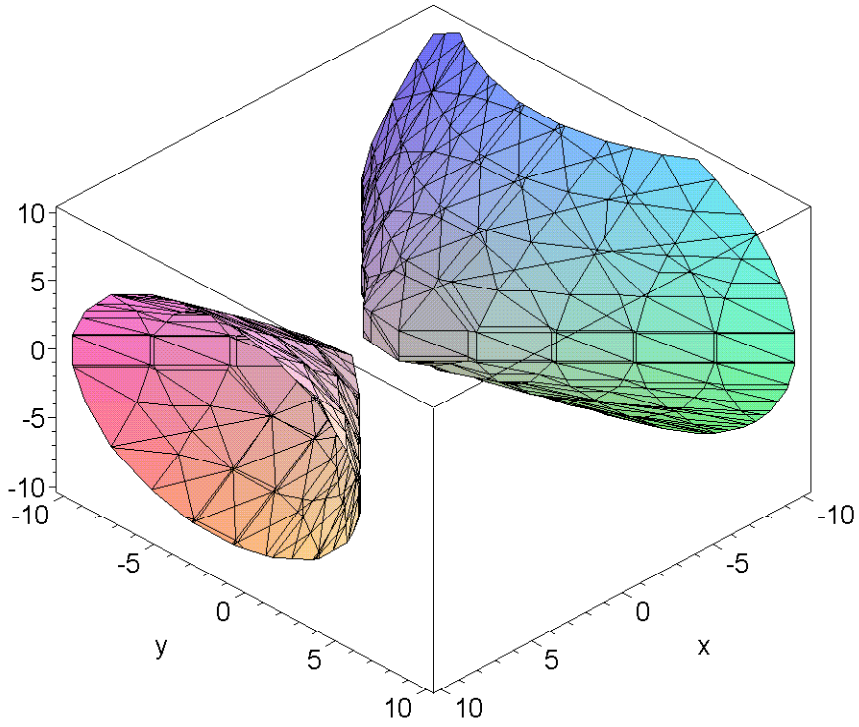
$$l := [1, 7 + t, 0]$$

next set

Sketch

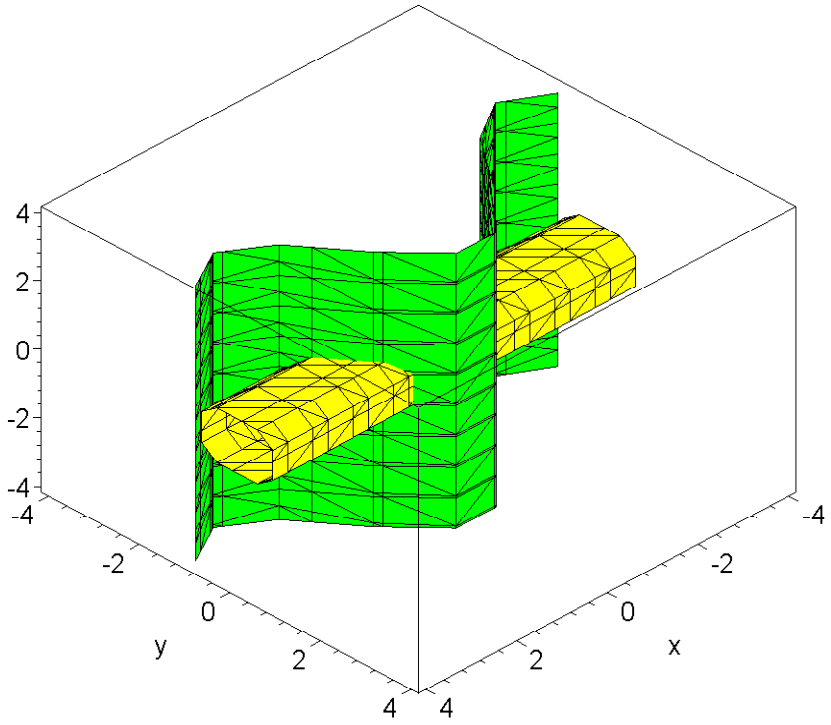
1

```
> implicitplot3d(x^2 -y^2- z^2 -4*x-4*y= 0,x=-10..10,y=-10..10, z=
  -10 .. 10, axes=boxed);
```



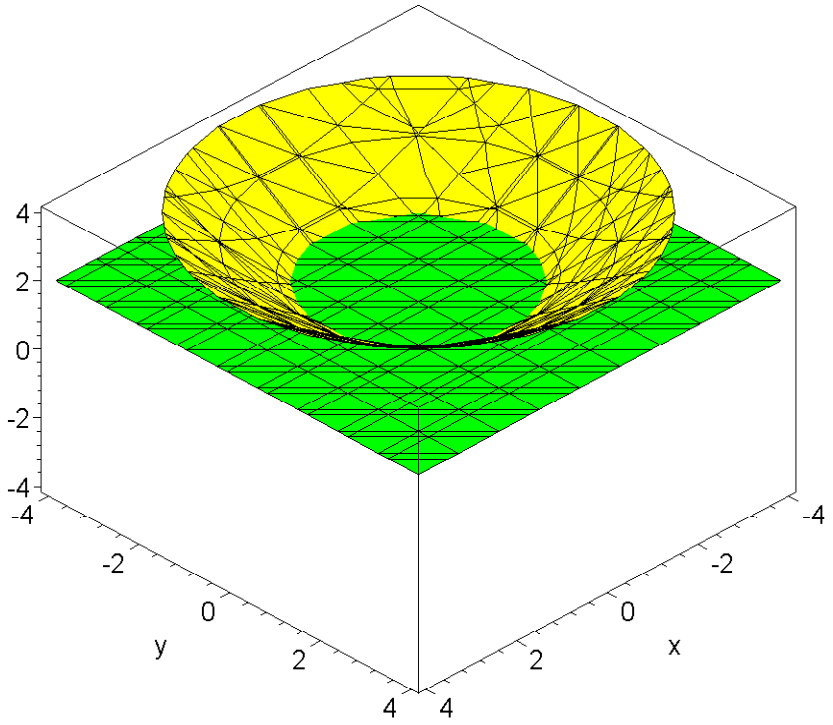
2

```
> plot1:=implicitplot3d( y^2+z^2=1, x = -4..4, y = -4 .. 4, z= -4 ..
  4, color=yellow): plot2:=implicitplot3d( y = sqrt(2)*cos(x), x =
  -4..4, y = -4 .. 4, z=-4 .. 4,color=green):

  display3d((plot1,plot2),axes=boxed);
>
```



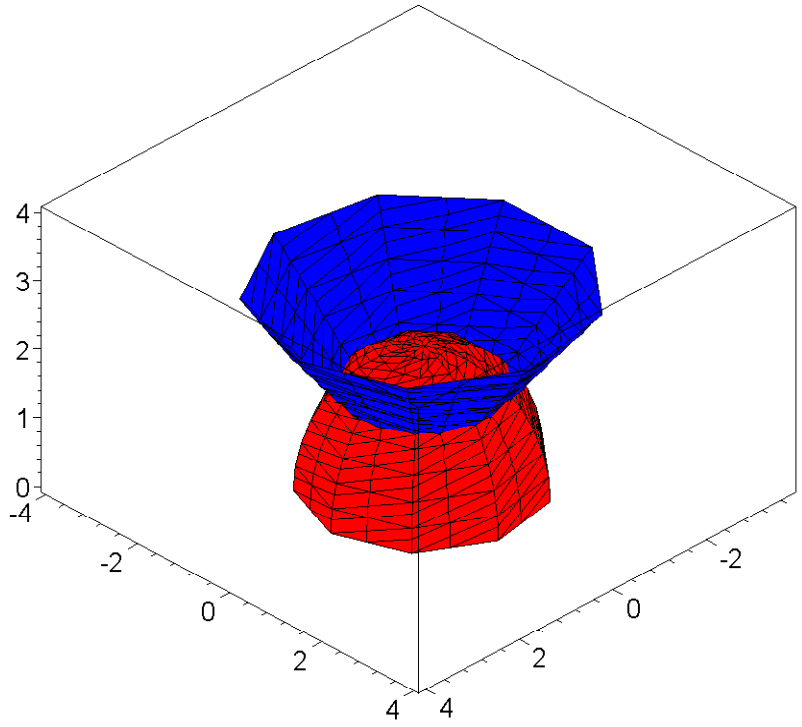
1

```
> plot1:=implicitplot3d( z=sqrt(x^2+y^2), x = -4..4, y = -4 .. 4, z=
  -4 .. 4, color=yellow): plot2:=implicitplot3d( z=2, x = -4..4, y =
  -4 .. 4, z=-4 .. 4,color=green):

  display3d((plot1,plot2),axes=boxed);
```



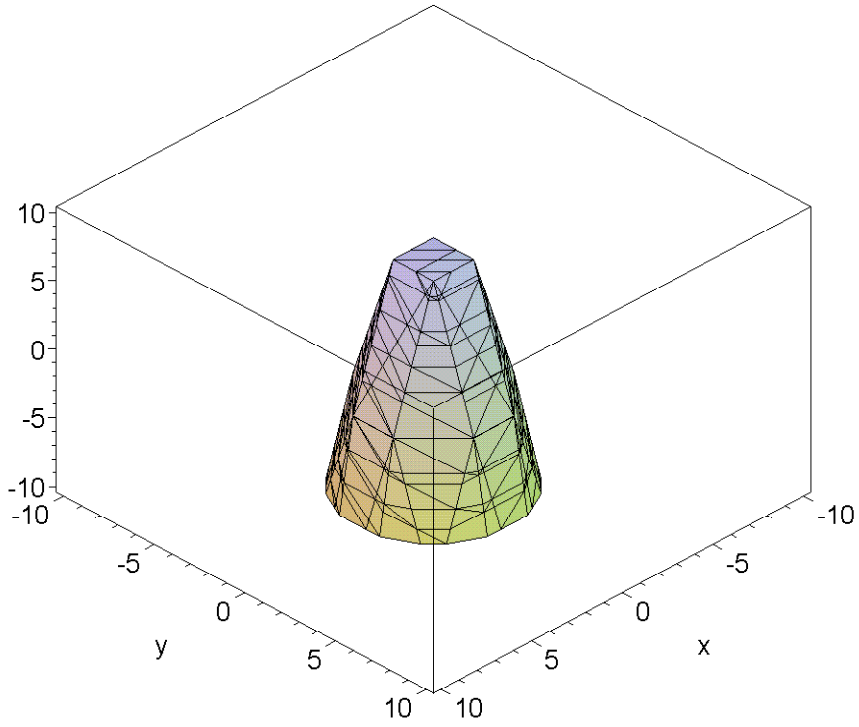
2

```
> plot1:=implicitplot3d( phi=Pi/4, rho = 0 ..4, theta = 0 .. 2*Pi,
  phi = 0 ..
  Pi/2,coords=spherical,color=blue):plot2:=implicitplot3d( rho=2,
  rho = 0 ..4, theta = 0 .. 2*Pi, phi = 0 ..
  Pi/2,coords=spherical,color=red):display3d((plot1,plot2),axes =
  boxed);
```



2.

```
> implicitplot3d(x^2 +y^2 +z = 9,x=-10..10,y=-10..10, z= -10 .. 10,
  axes=boxed);
>
```



IV 1. Find the equation of the plane .

```
> a:=[0,0,0];b:=[1,1,1]; c:=[1,2,3];#  points in the  plane
```

$$a := [0, 0, 0]$$

$$b := [1, 1, 1]$$

$$c := [1, 2, 3]$$

```
> ba := b-a; bc:= b-c;  n:=crossprod(ba,bc); # normal of 2 vectors
  in the plane
```

$$ba := [1, 1, 1]$$

$$bc := [0, -1, -2]$$

$$n := [-1, 2, -1]$$

```
> R:=vector([x,y,z]);
```

$$R := [x, y, z]$$

```
> u:=dotprod(R,n);k :=(dotprod(n,a)); # direction and konstant
```

$$u := -x + 2\,y - z$$

$$k := 0$$

```
> u = k; # equation of the  plane
```

$$-x + 2\,y - z = 0$$

2 and 3 clear

I 1a

```
> v1:=[0,0,1]; v2:=[0,2,0];v3:=[3,0,0];
```

$$v1 := [0, 0, 1]$$

$$v2 := [0, 2, 0]$$

$$v3 := [3, 0, 0]$$

Method 1 cross product

```
> a:= v1-v2; b=v2-v3; area := norm(crossprod(a,b),2)/2;
```

$$a := [0, -2, 1]$$

$$[1, 1, 1] = [-3, 2, 0]$$

$$area := \frac{1}{2}\sqrt{14}$$

Method 2 altitude method

```
> a:= v1-v2; b:=v3-v2; u:=(dotprod(a,b)/norm(b,2)*(b/norm(b,2)));
```

$$a := [0, -2, 1]$$

$$b := [3, -2, 0]$$

$$u := \left[\frac{12}{13}, \frac{-8}{13}, 0\right]$$

the altitude is

```
> alt := a-u;dotprod(alt,u);
>
```

$$alt := \left[\frac{-12}{13}, \frac{-18}{13}, 1\right]$$

$$0$$

and area is

```
> area := norm(alt,2) * norm(b,2)/2;
```

$$area := \frac{7}{2}$$

11 in a paralleogram the diagonal play a key role: they are perp; set d=[x,y} will be the 4th vertex

```
>  a:= [-1,2]; b:=[6,4];c:=[1,20]; d:=[x,y];
```

$$a := [-1, 2]$$

$$b := [6, 4]$$

$$c := [1, 20]$$

$$d := [x, y]$$

```
> bd := d-b; ca:=c-a;ad:=d-a;dc:=c-d; # thes are the digonals
```

$$bd := [-6 + x, -4 + y]$$

$$ca := [2, 18]$$

$$ad := [1 + x, -2 + y]$$

$$dc := [-x + 1, -y + 20]$$

```
> dotprod(bd,ca)=0; # must be zero so x+9y=42
```

$$-84 + 2x + 18y = 0$$

the second property is that the diagonal intersect at Po which bisects them, Po lies half way between A

and C

```
> Po:= (a+c)/2;# halfway and now it should be halfway on bd
```

$$Po := [0, 11]$$

```
> (b+Po)/2=(Po+d)/2;
```

$$\left[3, \frac{15}{2}\right] = \left[\frac{1}{2}x, \frac{1}{2}y + \frac{11}{2}\right]$$

so x=6 and y=4

2 just use the inner prodyct form of Ax(BxC)

3 Find L

```
> P:=[-1,-2,4]; Q:=[4,2,1]; v:=P-Q;
```

$$P := [-1, -2, 4]$$

$$Q := [4, 2, 1]$$

$$v := [-5, -4, 3]$$

```
> L:= t -> (t*v +P);
```

$$L := t \to t\,v + P$$

```
> x:=-5*t-1;y:=-4*t-2;z:=3*t+4;
```

$$x := -5\,t - 1$$

$$y := -4\,t - 2$$

$$z := 3\,t + 4$$

```
> eq := x+y+2*z=11;
> solve(eq,t); # answer in t
```

$$eq := -3\,t + 5 = 11$$

$$-2$$

```
> L(-2);# the point
```

$$[9, 6, -2]$$

4 is easy

5

```
> A:=[3,2,1]; v:=[-2,1,3];B:=[-2,3,1];R:=[x,y,z];
```

$$A := [3, 2, 1]$$

$$v := [-2, 1, 3]$$

$$B := [-2, 3, 1]$$

$$R := [-5\,t - 1, -4\,t - 2, 3\,t + 4]$$

```
> L:= t -> v*t + A;
```

$$L := t \to t\,v + A$$

A and B lie in the Plane so n is the cross product

```
> AB:= B- A;
```

$$AB := [-5, 1, 0]$$

```
> n:= crossprod(AB,v);
```

$$n := [3, 15, -3]$$

```
> dotprod(n,R)= dotprod(n,B); # the plane
```

```
>
```

$$-45 - 84\,\bar{t} = 36$$

6 idea is to find the line perp to the plane through the origin, get it's intersection P in the plane then measure that distance d(O,P) to the origin

```
> n:=[1,2,2]; OO:=[0,0,0]; L:= t -> t*n;
```

$$n := [1, 2, 2]$$

$$OO := [0, 0, 0]$$

$$L := t \to t\,n$$

```
> x:=1*t;y:=2*t;z:=3*t;
```

$$x := t$$

$$y := 2\,t$$

$$z := 3\,t$$

```
> eq := x+2*y+2*z=6;solve(eq,t);
```

$$eq := 11\,t = 6$$

$$\frac{6}{11}$$

```
> L(%); # the point  of intersection
```

$$\left[\frac{6}{11}, \frac{12}{11}, \frac{12}{11}\right]$$

```
> norm(%,2); # the answer
```

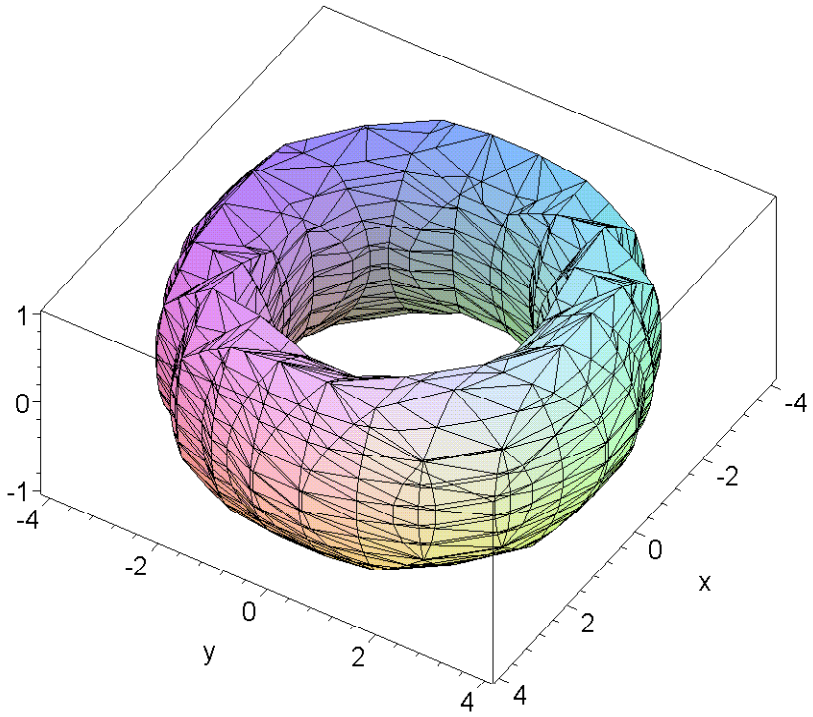
$$\frac{18}{11}$$

7b

```
> restart:with(plots):with(student):g:= (x,y,z) -> (
  sqrt(x^2+y^2)-3)^2+z^2 =1;
Warning, the name changecoords has been redefined
```

$$g := (x, y, z) \to \left(\sqrt{x^2 + y^2} - 3\right)^2 + z^2 = 1$$

```
> implicitplot3d( ((sqrt(x^2+y^2)-3)^2 + z^2 =1), x = -4 .. 4, y =
  -4 .. 4, z = -1  .. 1, axes=boxed);
```



8

```
> r:= t -> [t,1,arctan((t+1)/(1-t))];r0:=r(2);# the point
```

$$r := t \to \left[t,\ 1,\ \arctan\left(\frac{t+1}{1-t}\right) \right]$$

$$r0 := [2,\ 1,\ -\arctan(3)]$$

```
> diff(r(t),t);
```

$$\left[1,\ 0,\ \frac{\dfrac{1}{1-t} + \dfrac{t+1}{(1-t)^2}}{1 + \dfrac{(t+1)^2}{(1-t)^2}} \right]$$

```
> v:=eval(%,t=2);# the vector
```

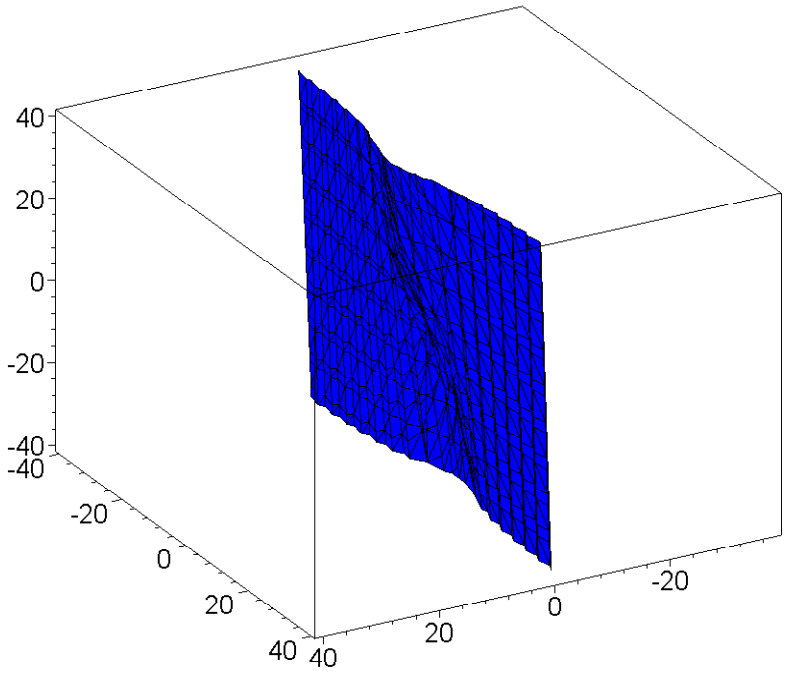
$$v := \left[1,\ 0,\ \frac{1}{5} \right]$$

```
> ans:= v*t+r0;
```

$$ans := \left[1,\ 0,\ \frac{1}{5} \right] t + [2,\ 1,\ -\arctan(3)]$$

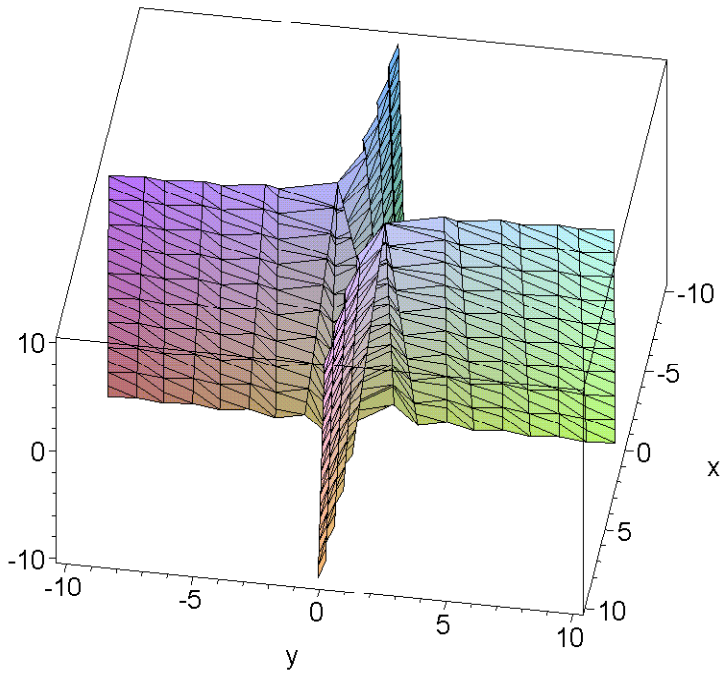
9

```
> implicitplot3d( z=r^2*cos(theta), r = 0 ..40, theta = 0 .. 2*Pi, z
  = -40 .. 40,coords=cylindrical,color=blue,axes=boxed);
```



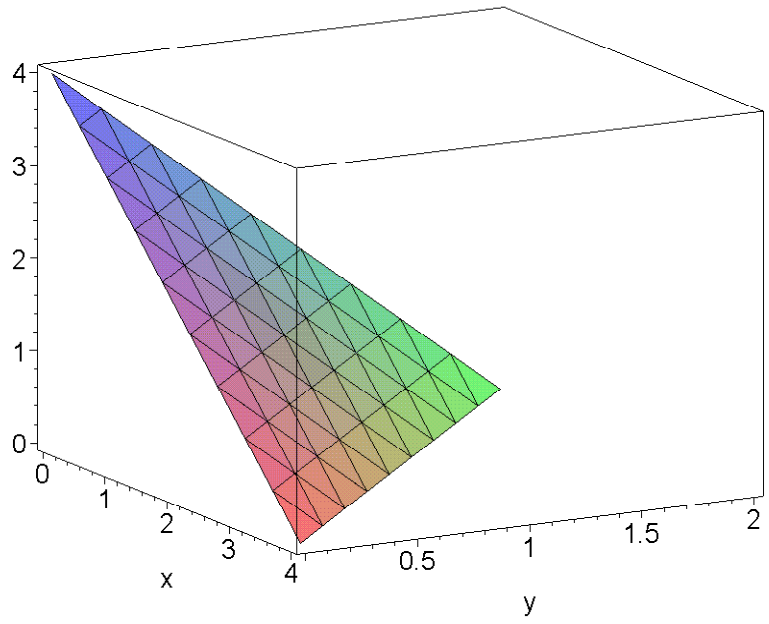
10

```
> implicitplot3d( z=8*x*y, x = -10 ..10, y = -10 .. 10, z = -10 ..
  10,axes=boxed);
```



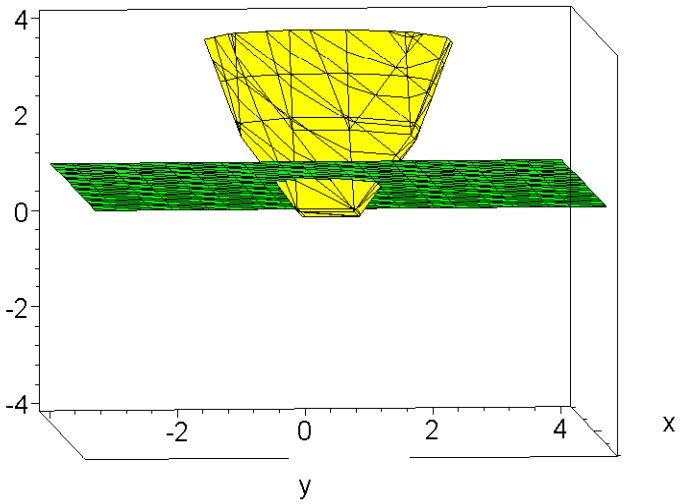
11

```
> implicitplot3d( x+2*y+z=4, x = 0 ..4, y = 0 .. 2, z = 0 ..
  4,axes=boxed);
>
```



12

```
> plot1:=implicitplot3d( z=(x^2+y^2), x = -4..4, y = -4 .. 4, z= -4
  .. 4, color=yellow): plot2:=implicitplot3d( z=1, x = -4..4, y = -4
  .. 4, z=-4 .. 4,color=green):

  display3d((plot1,plot2),axes=boxed);
```


4
2
0
-2
-4
-2
0
2
4
x
y

>