```
> restart:with( plots):with (student):
Warning, the name changecoords has been redefined
```

Section I ex 1

```
> g(x):= ln(sec(3*x)): Diff(g(x),x)= diff(g(x),x);
```

$$\frac{\partial}{\partial x}\ln(\sec(3x)) = 3\tan(3x)$$

ex 2

```
> g(x):= arcsin(1+ sin(x)): Diff(g(x),x)= diff(g(x),x);
```

$$\frac{\partial}{\partial x}\arcsin(1+\sin(x)) = \frac{\cos(x)}{\sqrt{1-(1+\sin(x))^2}}$$

ex 3

```
> g(x):= arctan(1/(x^2)): Diff(g(x),x)= diff(g(x),x):simplify(%);
```

$$\frac{\partial}{\partial x}\arctan\left(\frac{1}{x^2}\right) = -2\frac{x}{x^4+1}$$

ex 4

```
> g(x):= exp(1+(x^2)): Diff(g(x),x)= diff(g(x),x);
```

$$\frac{\partial}{\partial x}\mathbf{e}^{(1+x^2)} = 2x\,\mathbf{e}^{(1+x^2)}$$

ex 5

```
> g(x):= arcsin(ln(sqrt(x))): Diff(g(x),x)=
  diff(g(x),x):simplify(%);
>
```

$$\frac{\partial}{\partial x}\arcsin\left(\frac{1}{2}\ln(x)\right) = \frac{1}{x\sqrt{4-\ln(x)^2}}$$

ex 6

```
> g(x):= x^k*exp(-x): Diff(g(x),x)= diff(g(x),x):g1:=simplify(%);
```

$$g1 := \frac{\partial}{\partial x}x^k\,\mathbf{e}^{(-x)} = x^{(k-1)}\,k\,\mathbf{e}^{(-x)} - x^k\,\mathbf{e}^{(-x)}$$

ex 7

```
> g(x):= exp(-x)*arccos(x)/cos(x): Diff(g(x),x)=
  diff(g(x),x):g1:=simplify(%);
```

$g1 :=$

$$\frac{\partial}{\partial x}\frac{\mathbf{e}^{(-x)}\arccos(x)}{\cos(x)} = -\frac{(\arccos(x)\cos(x)\sqrt{1-x^2}+\cos(x)-\arccos(x)\sin(x)\sqrt{1-x^2})\,\mathbf{e}^{(-x)}}{\cos(x)^2\sqrt{1-x^2}}$$

Section II

ex 2

```
> g:= x -> (x^2+6)/x^2:
  Int(g(x),x)= int(g(x),x);
```

$$\int \frac{x^2+6}{x^2}\,dx = x - \frac{6}{x}$$

$$\int \sec(x) + \sin(x)^2\,dx = \ln(\sec(x)+\tan(x)) - \frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

ex 3

```
> g:= x -> 1/sqrt(3-x^2):
  Int(g(x),x)= int(g(x),x): simplify(%);
```

$$\int \frac{1}{\sqrt{3-x^2}}\,dx = \arcsin\left(\frac{1}{3}\sqrt{3}\,x\right)$$

```
>
```

ex 4

```
> g:= x -> x/(x^2+4):
  Int(g(x),x)= int(g(x),x);
```

$$\int \frac{x}{x^2+4}\,dx = \frac{1}{2}\ln(x^2+4)$$

ex 5

```
> g:= x -> 1/(x^2+4):Int(g(x),x)= int(g(x),x);
```

$$\int \frac{1}{x^2+4}\,dx = \frac{1}{2}\arctan\left(\frac{1}{2}x\right)$$

ex 6

```
> g:= x -> x/sqrt(1-x^2):
  Int(g(x),x)= int(g(x),x);
```

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2}$$

ex 7

```
> g:= x -> (ln(x))^3/x:
  Int(g(x),x)= int(g(x),x);
```

$$\int \frac{\ln(x)^3}{x}\,dx = \frac{1}{4}\ln(x)^4$$

ex 8

```
> g:= x -> (2*x+1)/(x^2+x+1):
  Int(g(x),x)= int(g(x),x):simplify(%);Int(g(x),x=0..1)=
  int(g(x),x=0..1);
```

$$\int \frac{2x+1}{x^2+x+1}\,dx = \ln(x^2+x+1)$$

$$\int_0^1 \frac{2x+1}{x^2+x+1}\,dx = \ln(3)$$

ex 9

```
> g:= x -> (ln(4*x^2-2*x+1)):
  Diff(Int(g(x),x),x)= simplify(diff(int(g(x),x),x));
```

$$\frac{\partial}{\partial x}\int \ln(4x^2-2x+1)\,dx = \ln(4x^2-2x+1)$$

ex 10

```
> g:= x -> (exp(-x)-exp(x)):
  Int(g(x),x)= int(g(x),x);
```

$$\int \mathbf{e}^{(-x)} - \mathbf{e}^{x}\,dx = -\mathbf{e}^{(-x)} - \mathbf{e}^{x}$$

ex 11

```
> g:= x -> 1/(sqrt(x)*exp(sqrt(x))):
  Int(g(x),x)= int(g(x),x); Int(g(x),x=0..1)= int(g(x),x=0..1);
```

$$\int \frac{1}{\sqrt{x}\,\mathbf{e}^{(\sqrt{x})}}\,dx = -2\,\frac{1}{\mathbf{e}^{(\sqrt{x})}}$$

$$\int_0^1 \frac{1}{\sqrt{x}\ \mathbf{e}^{(\sqrt{x})}}\,dx = -2\,\mathbf{e}^{(-1)} + 2$$

ex 14

```
> simplify(cos(arctan(1)));
```

$$\frac{1}{2}\sqrt{2}$$

ex 15

```
> g:= x -> 1/(4+x^2):
  Int(g(x),x)= int(g(x),x); Int(g(x),x=-2..0)= int(g(x),x=-2..0);
```

$$\int \frac{1}{x^2+4}\,dx = \frac{1}{2}\arctan\left(\frac{1}{2}x\right)$$

$$\int_{-2}^{0} \frac{1}{x^2+4}\,dx = \frac{1}{8}\pi$$

ex 16

```
> g:= x -> exp(tan(x))*(sec(x)*sec(x)):
  Int(g(x),x);
```

$$\int \mathbf{e}^{\tan(x)}\sec(x)^2\,dx$$

ex 17

```
> g:= x -> 1/sqrt(4-x^2):
  Int(g(x),x)= int(g(x),x); Int(g(x),x=1..sqrt(3))=
  int(g(x),x=1..sqrt(3));
```

$$\int \frac{1}{\sqrt{4-x^2}}\,dx = \arcsin\left(\frac{1}{2}x\right)$$

$$\int_1^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\,dx = \frac{1}{6}\pi$$

ex19

```
> g(x):= x^(ln(x)): Diff(g(x),x)= diff(g(x),x);
```

$$\frac{\partial}{\partial x} x^{\ln(x)} = 2\,\frac{x^{\ln(x)}\ln(x)}{x}$$

ex 20

```
> g(x):= 4^(3*x): Diff(g(x),x)= diff(g(x),x);
```

$$\frac{\partial}{\partial x} 4^{(3x)} = 3\,4^{(3x)}\ln(4)$$

ex 21

```
> g(x):= 4*x*exp(x): Diff(g(x),x)=
  diff(g(x),x);solve(diff(g(x),x)=0);
```

$$\frac{\partial}{\partial x}(4\,x\,\mathbf{e}^{x}) = 4\,\mathbf{e}^{x} + 4\,x\,\mathbf{e}^{x}$$

$$-1$$

ex 22

```
> g:= x -> 1/(x*(ln(x))^n);
  Int(g(x),x)= int(g(x),x); Int(g(x),x=exp(1)..exp(2))=
  int(g(x),x=exp(1)..exp(2));
```

$$g := x \to \frac{1}{x\,\ln(x)^{n}}$$

$$\int \frac{1}{x\,\ln(x)^{n}}\,dx = -\frac{\ln(x)}{(-1+n)\,\mathbf{e}^{(n\ln(\ln(x)))}}$$

$$\int_{\mathbf{e}}^{\mathbf{e}^{2}} \frac{1}{x\,\ln(x)^{n}}\,dx = \frac{-2+2^{n}}{2^{n}\,(-1+n)}$$

ex 23

```
> g:= x -> 1/sqrt(a^2-x^2);Int(g(x),x)= int(g(x),x);
  f:= x -> x/sqrt(a^2-x^2);Int(f(x),x)= int(f(x),x);
  g1:=int(g(x),x=0..a);f1:=int(f(x),x=0..a);
> solve(g1=f1,a);
```

$$g := x \to \frac{1}{\sqrt{a^{2}-x^{2}}}$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right)$$

$$f := x \to \frac{x}{\sqrt{a^2 - x^2}}$$

$$\int \frac{x}{\sqrt{a^2 - x^2}}\, dx = -\sqrt{a^2 - x^2}$$

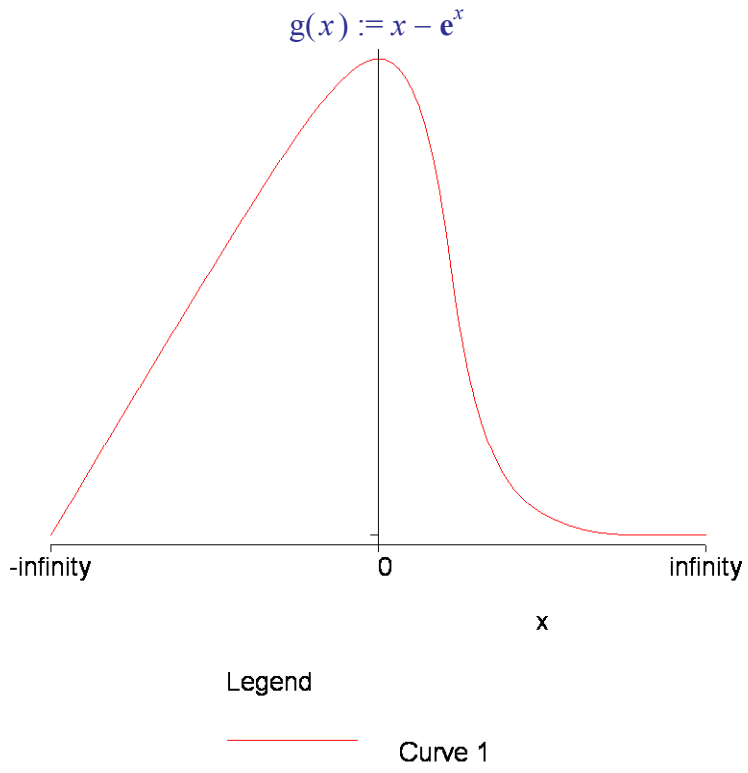
$$g1 := \frac{1}{2}\frac{\pi}{\sqrt{a^2}\sqrt{\frac{1}{a^2}}}$$

$$f1 := \sqrt{a^2}$$

$$\frac{1}{2}\pi, -\frac{1}{2}\pi$$

Section III ex1

```
> g(x) := x -exp(x);plot(g(x),x=-infinity .. infinity);
```

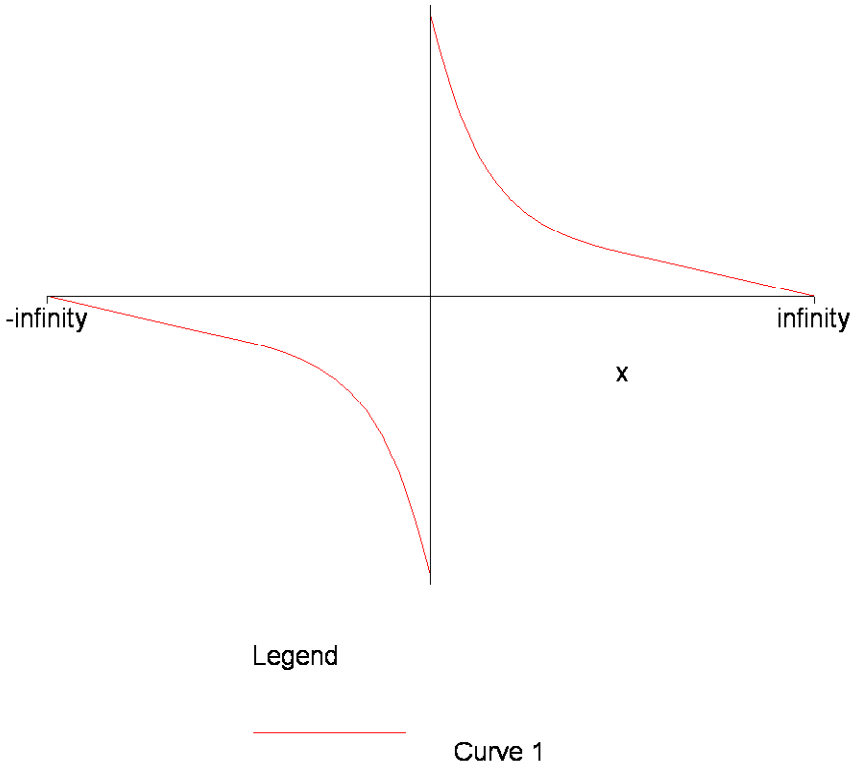


>

ex2

```
> g(x) := arctan(1/x);plot(g(x),x=-infinity..infinity);
```

$$g(x) := \arctan\left(\frac{1}{x}\right)$$



>

Section II

ex1

```
> g(x):= (exp(4*x)-1)/sin(x);
```

$$g(x) := \frac{e^{(4x)} - 1}{\sin(x)}$$

```
> Limit(g(x), x=0)=limit(g(x), x=0);
```

$$\lim_{x \to 0} \frac{e^{(4x)} - 1}{\sin(x)} = 4$$

ex2

```
> g(x):= (1 -2*x)^(1/x);
```

$$g(x) := (1 - 2x)^{\left(\frac{1}{x}\right)}$$

```
> Limit(g(x), x=0, right)=limit(g(x), x=0, right);
```

$$\lim_{x \to 0+} (1 - 2x)^{\left(\frac{1}{x}\right)} = \mathbf{e}^{(-2)}$$

ex3

```
> g(x):= x^(sin(x));
```

$$g(x) := x^{\sin(x)}$$

```
> Limit(g(x), x=0, right)=limit(g(x), x=0, right);
```

$$\lim_{x \to 0+} x^{\sin(x)} = 1$$

ex4

```
> g(x):= x/(x+1);
```

$$g(x) := \frac{x}{x+1}$$

```
> Limit(g(x), x= infinity)=limit(g(x), x=infinity);
```

$$\lim_{x \to \infty} \frac{x}{x+1} = 1$$

ex5

```
> g(x) := ln(Pi/x) / tan(x):
> Limit (g(x), x = Pi)= limit(g(x),x = Pi);
```

$$\lim_{x \to \pi} \frac{\ln\left(\frac{\pi}{x}\right)}{\tan(x)} = -\frac{1}{\pi}$$

ex1

```
> g:= x -> x*e^(-x):
  Int(g(x),x)= int(g(x),x);
```

ex2

```
> g:= x -> ln(x):
  Int(g(x),x)= int(g(x),x);
```

ex3

```
> g:= x -> x*sin(x):
  Int(g(x),x)= int(g(x),x);
>
```

$$\int x\, e^{(-x)}\, dx = -\frac{x\, \mathbf{e}^{(-x \ln(e))}}{\ln(e)} - \frac{\mathbf{e}^{(-x \ln(e))}}{\ln(e)^2}$$

$$\int \ln(x)\, dx = x \ln(x) - x$$

$$\int x \sin(x)\, dx = \sin(x) - x \cos(x)$$

```
>
```

[>
[>